PROBLEM 1 - BEAM BLOCKAGE IN complex TERRAIN

..... REFER TO MAP .....

...... BEAM GEOMETRY PLOTS ARE INADEQUATE HERE .....

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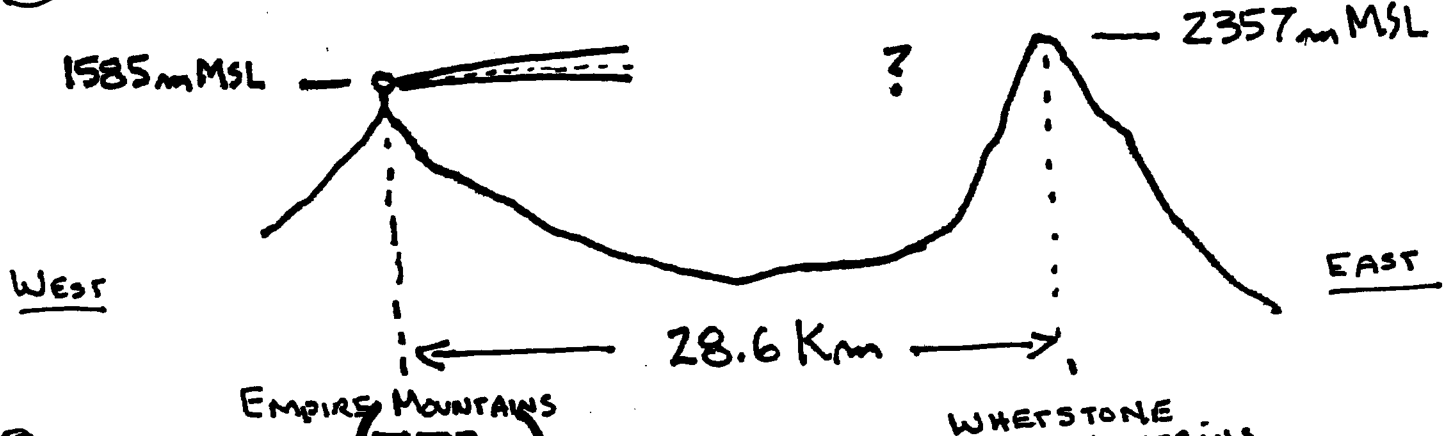

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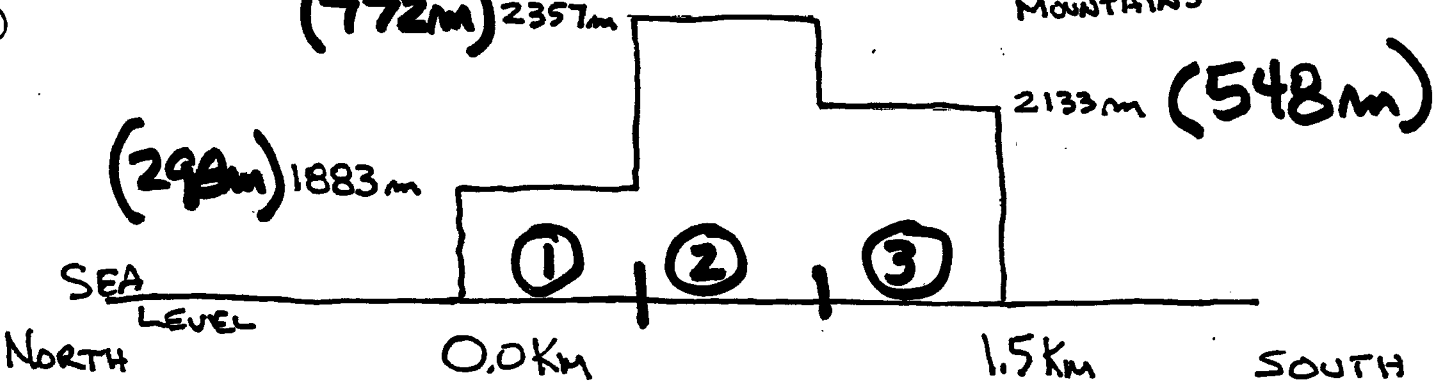
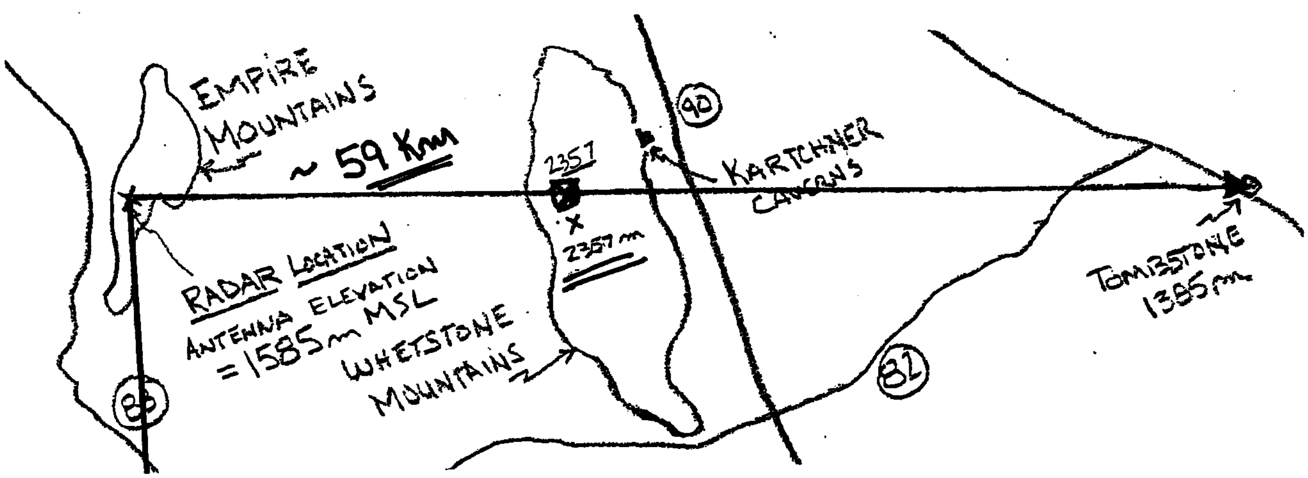


<u>NOTE</u> sketches a & b are <u>NOT</u> to scale

<u>Assumptions</u> $\Delta N / \Delta h = -39$ N-UNITS/Km

RANGE TO WHETSTONE MOUNTAINS IS CONSTANT IN SKETCH b

RADAR IS SCANNING AS PER WSR-88D VCP-11



Problem 1

1- How many 1° BEAM radials are there from 0 to 1.5 Km? Sketch their positions the sea level line and number them from North to South. Edge (northern) of radial #1 is positioned exactly at 0.0 Km

Ans - Three 1° beam radials strike the portion of the Whetstone Mountains that we are considering - see diagram.

As per BEAM width (diameter) =

Θ(radians) · r(ange) or

$0.0175 \cdot 28.6\,\text{Km} = 0.5\,\text{Km} \times 3 = 1.5\,\text{Km}$

2- Calculate the % of the beam that is blocked by the terrain of the Whetstone Mountains as sketched above. Do this for each numbered radial for elevation tilts 1 through 3.

Ans - We need to determine the positions, i.e., the beam geometry, for tilts 1-3, along radials 1-3, at the range of the Whetstone Mountains.

First step here is to adjust the MSL elevations of the Whetstones to be relative to the elevation of the radar antenna — as per sketch.

Then use eq. 3.12 to compute the height of the beam centers AT THE RANGE of the Whetstones:

$$3.12 \quad H = \left(r^2 + R'^2 + 2rR'\sin\phi\right)^{\frac{1}{2}} - R' + H_0$$

Problem 1

REMEMBER: This eq. is only valid if standard refraction applies - as per, $\Delta N/\Delta H = -39$ N-UNITS/KM

ϕ (IN THIS EQ!) = the elevation Angles of the first 3 radar tilts - as per beam geometry figures for WSR-88D. Tilt 1: $\phi = 0.5°$ Tilt 2: $\phi = 1.45°$ Tilt 3: $\phi = 2.4°$

$R' = 4/3 R$ where $R = 6374$ Km

H_o = elevation of Antenna = 0 here since mountain elevations have been adjusted to be relative to H_o

SO: $R' = 8.498667 \times 10^6 m$

$R'^2 = 7.222734 \times 10^{13} m^2$

$r^2 = (28.6 \times 10^3 m)^2 = 8.179600 \times 10^8 m^2$

$2rR' = 4.861238 \times 10^{11} m^2$

Tilt 1	$\sin(0.5) =$	8.726535×10^{-3}
Tilt 2	$\sin(1.45) =$	2.530457×10^{-2}
Tilt 3	$\sin(2.4) =$	4.187565×10^{-2}

Computed BEAM CENTERS ARE:

$H_{0.5°} = 298 m$

$H_{1.45°} = 772 m$

$H_{2.4°} = 1{,}246 m$

PROBLEM 1

Sample calculation for tilt 1:

$$H = \left[(8.179600 \times 10^{8}\,m^2) + (7.222734 \times 10^{13}\,m^2) + (4.861238 \times 10^{11}\,m^2)(8.7265 \times 10^{-3})\right]^{1/2}$$

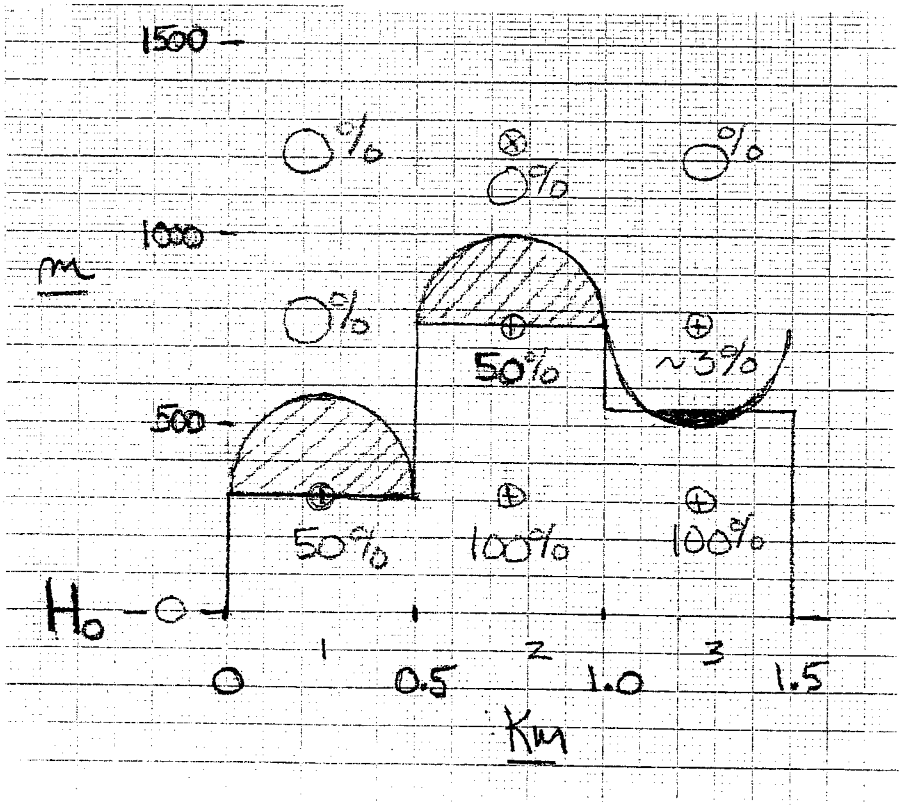
$$- 8.49867 \times 10^{6}\,m$$

$$\begin{array}{rl}
.000081796000 & \times 10^{13}\,m^2 \\
+\ 7.222734000000 & \times 10^{13}\,m^2 \\
+\ .000424217640 & \times 10^{13}\,m^2 \\
\hline
\end{array}$$

$$(7.2232400136\,4 \times 10^{13}\,m^2)^{1/2} =$$

$$\begin{array}{rl}
8.49896465 & \times 10^{6}\,m \\
-\ 8.49866700 & \times 10^{6}\,m \\
\hline
.00029765 & \times 10^{6}\,m
\end{array}$$

$H = 298\,m$



Problem 1

3- THE highest TERRAIN SKETCHED ABOVE IS located BETWEEN THE RADAR AND TOMBSTONE. CALCULATE THE HEIGHT, OVER TOMBSTONE, OF THE BEAM CENTER OF THE LOWEST ELEVATION TILT THAT has NO TERRAIN BLOCKAGE.

CONSIDER

r to TOMBSTONE ≅ 59 Km
elevation of TOMBSTONE Relative to radar antenna is
MINUS 200m

use eq. 3.12 again for 3 elevation tilt (2.4°) since this is lowest, completely unblocked tilt

CENTER of 3rd tilt is 2675m above elevation of radar antenna OR 2875m above TOMBSTONE.

PROBLEM 2 – POINT TARGETS

.....REFER TO MAP....

A MAGNIFICENT Golden EAGLE IS SOARING ACROSS THE SKIES of southeastern ARIZONA, HEADED FOR ITS NESTING GROUNDS AT THE PATAGONIA/SONOITA CREEK Wildlife Preserve. Although the bird is a great flyer, IT CAN NOT CLIMB HIGHER than 5 Km MSL.

THE AVERAGE RADAR CROSS-SECTION FOR THIS EAGLE IS $1\ m^2$ AT S-BAND WAVELENGTH. $\Delta N/\Delta H = -39$ N-UNITS/Km everywhere

1- IS THE EAGLE A Mie or OPTICAL SCATTER? BE SURE to SHOW your calculations.

Need to compute D/λ for the eagle using the effective Cross-section area. So:

$$\pi\left(\frac{D}{2}\right)^2 = 1\ m^2 = 1\times10^4\ cm^2$$

$$D^2 = (4/\pi)\times10^4\ cm^2$$

$$D^2 = 1.2732\times10^4\ cm^2$$

$$D = 1.13\times10^2\ cm^2$$

So that $D/\lambda \cong 11.3$ indicating that the eagle is a LARGE or OPTICAL scatterer

refer to text p. 69-72

PROBLEM 2

OR You could compute the effective circumference of the eagle and divide by wavelength to determine where the eagle falls on Fig. 4.2 in text, i.e., $\pi D/\lambda \cong 35.4$

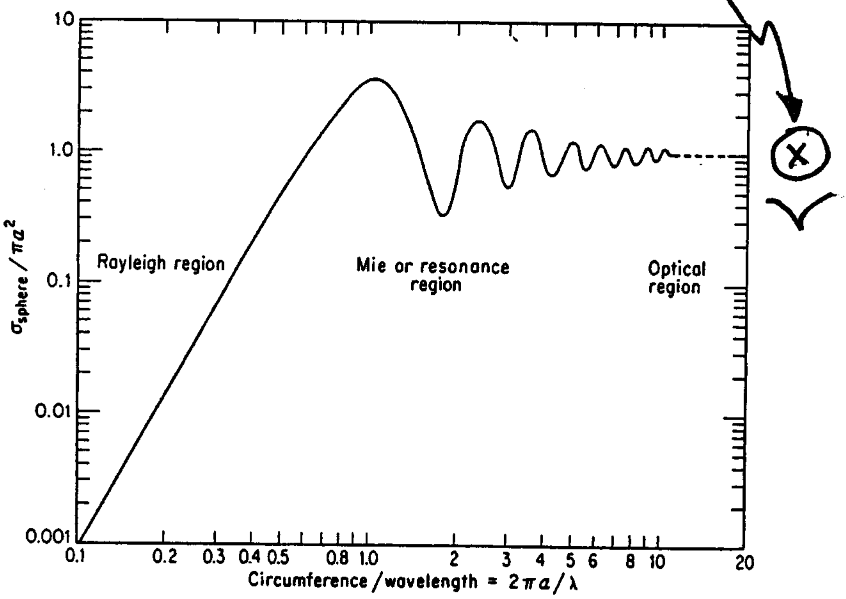


Figure 4.2 Normalized backscattering cross-sectional area of a sphere as a function of circumference normalized by radar wavelength λ. a = radius. From Skolnik, 1980, Introduction

2- All calculations are valid ONLY if eagle is flying ....

EXACTLY AT THE CENTER OF THE RADAR BEAM

refer to text p.67, p. 69, p.77-78

Problem 2

3- Calculate the maximum range at which our radar, if it were operating at sea level, could detect the eagle.

This question is essentially similar to the Airplane example that was worked out in the text, p. 77.
Use eq. 4.7 for point targets

4.7 $$P_r = \frac{P_t g^2 \lambda^2 \sigma}{64 \pi^3 r^4} \quad \text{or}$$

$$r = \left(\frac{P_t g^2 \lambda^2 \sigma}{P_{r_{min}} 64 \pi^3} \right)^{1/4}$$ where P_{r_m} = minimum detectable signal

$$r = \left(7956.97 \times 10^{20}\right)^{1/4} \; m^4 \frac{mW}{mW}$$

$r =$ 944.5 Km But, whoops, the eagle can't begin to fly high enough to be detected at this range!

Thus, defaulting to WSR-88D beam geometry plot, we get ~ 225 Km

Problem 2

4- Calculate the power returned to our radar operating on the Empire Mountains as the eagle soars over the P/S Creek Wildlife Preserve.

Note: Since r, range, to the preserve is ~42.5 Km and since eagle can attain heights up to 3415 m above the elevation of the radar antenna, it could possibly be detected in 5 different elevation tilts.

To do the calculation, we need to use eq. 4.7 again

4.7 $$P_r = \frac{P_t g^2 \lambda^2 \sigma}{64 \pi^3 r^4}$$

$$P_r = \frac{49.928 \times 10^{14} \text{ mW m}^4}{6.4728 \times 10^{21} \text{ m}^4}$$

$$P_r = 7.714 \times 10^{-7} \text{ mW}$$

5- Calculate the power returned to our radar operating on the Empire Mountains when the eagle began its flight home from high above Organ Pipe National Monument – NOTE: the range from our radar to Organ Pipe is 200 Km.

THE FIRST ISSUE which MUST BE ADDRESSED HERE IS whether or NOT the eagle CAN fly high enough out at Organ Pipe to reach the Center of the 0.5° 1st tilt. Using the WSR-88D Beam Geometry plot we see that the beam Center of 1st tilt at 200 Km is A BIT higher than 4 Km above the elevation of the Antenna, BUT eagle CAN ONLY reach 3.415 Km

So eagle CAN NOT BE DETECTED AND $P_r \cong 0$

PROBLEM 3 - DISTRIBUTED TARGETS

FIRST, THROW THE HANDOUT FROM LAST CLASS - i.e., 21 Feb 2003 CLASS MEETING #16 - AWAY! I GOOFED UP THE CALCULATIONS AND WE'LL TRY TO GET THEM RIGHT IN THIS PROBLEM.

1 - For our radar, calculate the radar constants C_1, C_2, & C_3 be sure to show units in your answers and calculations

Note: $C_2 = C_1 |K|^2$ where $|K|^2$ is value for water drops

$C_3 = \frac{1}{C_2}$

From eq. 5.12 in text

$$C_1 = \left[\frac{\pi^3 g^2 \theta \phi h}{1024 \ln(2)}\right]\left(\frac{P_t}{\lambda^2}\right)$$

$$= \frac{(3.10\times10^1)(9.986\times10^8)(3.0625\times10^{-4})(1\times10^3 m)(5\times10^8 mW)}{(7.098\times10^2)(1\times10^{-2} m^2)}$$

$$\boxed{C_1 = 6.6783\times10^{17} mW/m}$$

$$C_2 = C_1(0.93) = 6.2108\times10^{17} mW/m$$

$$C_3 = \frac{1}{C_2} = 1.6101\times10^{-18} m/mW$$

PROBLEM 3

2- CALCULATE THE EFFECTIVE SAMPLE VOLUME, V, AT A RANGE OF 200 KM

from eq. 5.2

$$V = \pi \frac{r\theta}{2} \frac{r\phi}{2} \frac{h}{2}$$

$$V = \pi \left(\frac{2\times10^5 m \cdot 1.75\times10^{-2}}{2} \right)^2 \left(\frac{1\times10^3 m}{2} \right)$$

$$\underline{V = 4.8114\times10^9 m^3 \text{ or } 4.8114 \ Km^3}$$

3- IF THERE ARE 300 IDENTICAL SPHERICAL DROPS OF DIAMETER 1mm per m^3 distributed through V at 200 KM range, calculate the RADAR REFLECTIVITY FACTOR, Z.

from eq. 5.11

$$Z = \sum_{\text{unit vol}} D^6 = \frac{300 \cdot (1mm)^6}{m^3} = 300 \frac{mm^6}{m^3}$$

PROBLEM 3

4- CALCULATE THE P_r for above CONDITIONS IF THE SPHERICAL DROPS ARE COMPOSED ENTIRELY OF WATER.

from eq. 5.15

$$P_r = \frac{c_1 |K|^2 z}{r^2}$$

$$P_r = \frac{(6.6783 \times 10^{17}\ \frac{mW}{m})(0.93)(3 \times 10^{2}\ \frac{mm^6}{m^3})}{(2.0 \times 10^5\ m)^2}$$

$$= 4.6581 \times 10^{9}\ mW\ \frac{mm^6}{m^6}\left(10^{-18}\ \frac{m^6}{mm^6}\right)$$

$$\underline{P_r = 4.6581 \times 10^{-9}\ mW} \quad \text{for Water Drops}$$

5- WHAT WOULD P_r BE IF DROPS ARE ENTIRELY ICE?

$$P_r = \frac{(6.6783 \times 10^{17}\ \frac{mW}{m})(0.197)(3 \times 10^{2}\ \frac{mm^6}{m^3})}{4.0 \times 10^{10}\ m^2}\left(10^{-18}\ \frac{m^6}{mm^6}\right)$$

$$\underline{P_r = 9.8672 \times 10^{-10}\ mW} \quad \text{for Ice spheres}$$

Problem 3

6- WHAT IS THE LOGARITMIC RADAR REFLECTIVITY FACTOR, Z, FOR EACH SCENARIO, i.e., for #s 4 & 5?

for the water drops $\eta = 300 \frac{mm^6}{m^3}$ So that

$$Z = 10 \log_{10}\left(\frac{300 \frac{mm^6}{m^3}}{1 \frac{mm^6}{m^3}}\right) = \underline{24.8 \text{ dBZ}}$$

But for $\underline{\underline{ice}}$ we must find the equivalent η for water that matches the power returned for the ice sphere, so from eq. 5.15

$$\eta_e = \frac{r^2 P_r}{C_1 |K|^2_{water}}$$

$$\eta_e = \frac{(4 \times 10^2 m^2)(9.8672 \times 10^{-10} mW)}{(6.6783 \times 10^{7} mW/m)(0.93)} \left(10^{18} \frac{mm^6}{m^6}\right)$$

So $\eta_e = 6.3549 \times 10^1 \frac{mm^6}{m^3}$ and

$$Z_e = 10 \log_{10}\left(\frac{63.549 \frac{mm^6}{m^3}}{1 \frac{mm^6}{m^3}}\right) = \underline{18.0 \text{ dBZ}}$$

<u>5</u> <u>BONUS</u> <u>POINTS</u> - IDENTIFY THE MISTAKES I MADE ON FRIDAY'S HANDOUT.

Handout page 1 λ incorrectly given as $1 \times 10^{-2} m$

Page 3 & 4 units conversion written wrong

Problem 4 - Distributed Targets

LET'S CONSIDER A RADAR THAT'S SLIGHTY DIFFERENT THAN OUR HYPOTHETICAL RADAR - AS PER: ALL PARAMETERS SIMILAR EXCEPT

$$\lambda = 3\,cm \quad \text{AND} \quad \theta = \phi = 0.5 \text{ degree}$$

1- COMPARE THE EFFECTIVE ANTENNA AREAS FOR THE TWO RADARS.

USE eq. 4.5 $\quad A_e = \frac{g\lambda^2}{4\pi}$

for 10cm radar $\quad A_e = \frac{(3.16\times10^4)(1\times10^{-1}m)^2}{4\pi} = \underline{\underline{25.15\,m^2}}$

for 3 cm radar $\quad A_e = \frac{(3.16\times10^4)(3\times10^{-2}m)^2}{4\pi} = \underline{\underline{2.26\,m^2}}$

2- CALCULATE THE LOGRITHMIC RADAR CONSTANT, C_3, FOR THIS 3cm WAVELENGTH RADAR, as per eq. 5.19

$$Z = C_3 + P_r + 20\log_{10}(r)$$

Since $C_3 = 10\log_{10} c_3$ (See text p.94) we must first calculate c_3 for our 3cm radar with $\theta = \phi = 0.5°$

from eq 5.12

$$c_1 = \frac{(\pi^3)(3.16\times10^4)^2(8.75\times10^{-3})^2(1\times10^3 m)(5\times10^8 mW)}{[1024\ \ln(2)](3\times10^{-2}m)^2}$$

PROBLEM 4

So $C_1 \cong 1.8549 \times 10^{18} \frac{mW}{m}$

$C_2 = C_1 |K|^2$ for WATER DROPS $= 1.725 \times 10^{18} \frac{mW}{m}$

$C_3 = 1/C_2 = 5.797 \times 10^{-19} \frac{m}{mW}$ But consider eq. 5.9 and note that C_3 will need to consider units conversions so that

$$C_3 = 5.797 \times 10^{-19} \frac{m}{mW} \left(10^{18} \frac{mm^6}{m^6}\right)\left(\frac{10^6 m^2}{Km^2}\right)$$ and P_r in mW

$C_3 = 5.797 \times 10^5 \frac{mm^6}{m^3}$ where the units are for entire RHS of eq 5.9, so the logarithmetic form of C_3 is

$$10 \log_{10}\left(\frac{5.797 \times 10^5 \, mm^6/m^3}{1 \, mm^6/m^3}\right) = 10(5.763) \text{ or}$$

round to $\underline{C_3 \cong 58}$ dBZ

3- Show A UNITS ANALYSIS of eq. 5.19 to VERIFY that Z IS INDEED IN UNITS of dBZ

See above - key to correct units is remembering that terms on right hand side of eq. 5.9 are actually being multiplied so that appropriate conversion factors must be used so that final result has dBZ ≡ dBZ. An example in text would have helped.

PROBLEM 4

4- IF dBZ is 50 FOR A TARGET AT RANGE OF 100 KM, USE eq. 5.19 to CALCULATE P_r for THE TARGET.

use 5.9 with our radars C_3:

$$50\,dBZ = (58 + P_r + 40)\,dBZ$$

$P_r = -48$ dBm or

$$10^{\frac{-48}{10}} = 1.585 \times 10^{-5}\,mW$$

5- Calculate η, RADAR REFLECTIVITY, FOR this TARGET.

for this target we know that:

$$Z = 10^{\frac{50}{10}} = 1 \times 10^{5}\,\frac{mm^6}{m^3}$$ so that eq. 5.13 gives

$$\eta = \frac{\pi^5 |K|^2 (1 \times 10^5\,\frac{mm^6}{m^3})\,(10^{-18}\,m^6/mm^6)}{(3 \times 10^{-2}\,m)^4}$$

$$\underline{\eta = 3.51 \times 10^{-5}\,m^{-1}}$$

6- LIST THE ASSUMPTIONS YOU MADE TO DO these CALCULATIONS.

We have assumed spherical Rayleigh scatterers, uniformly distributed through radar sample volume, scatterers are water droplets, and we have ignored attenuation.

PROBLEM 5 - Doppler VELOCITY

1- FOR BOTH OF OUR hypothetical Doppler RADARS, i.e. the 10 & 3cm versions, operating at a PRF of 500/s, DETERMINE THE MAXIMUM UNAMBIGUOUS RANGES AND RADIAL VELOCITIES.

Use eq. 6.9 $\quad V_{r_{max}} = \dfrac{PRF\,\lambda}{4}$

for PRF 500/s $\quad V_{r_{max}} =$ 12.5 m/s @ 10 cm λ AND 3.75 m/s @ 3 cm λ

then use eq. 6.11 $\quad r_{max} = \dfrac{c}{2\,PRF}$

for PRF 500/s $\quad r_{max} =$ 300 Km for both wavelengths

2- DETERMINE THE SAME FOR BOTH RADARS operating At 1500/s PRF.

When we increase the PRF to 1500/s

then we find that

$V_{r_{max}} =$ 37.5 m/s @ 10 cm λ

and $\quad V_{r_{max}} =$ 11.25 m/s @ 3 cm λ

But for PRF 1500/s, the value of r_{max} becomes 100 Km for both radars

PROBLEM 5

3 - AT PRF 1500/s, DETERMINE THE RADIAL VELOCITY OF AN INBOUND TARGET, FOR BOTH RADARS, that PRODUCES A PHASE SHIFT OF EXACTLY $2\pi/3$ RADIANS

If AN INBOUND velocity (i.e., V_r is negative) produces a detected phase shift of $+2\pi/3$, then the velocity MUST BE FOLDED (exceeds the maximum unambiguous velocity). So the answer for the folded velocity (1st possible value) will be $-V_{r_{max}} + V_r$ which produces a phase shift of $-\pi/3$.

Use eq. 6.2

$$\frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dr}{dt} \qquad \phi = \text{phase} \qquad \frac{dr}{dt} = V_r$$

for $d\phi = -\pi/3$ and $dt = \frac{1}{PRF} = \frac{1}{1500\,s^{-1}} = 6.667\times10^{-4}\,s$

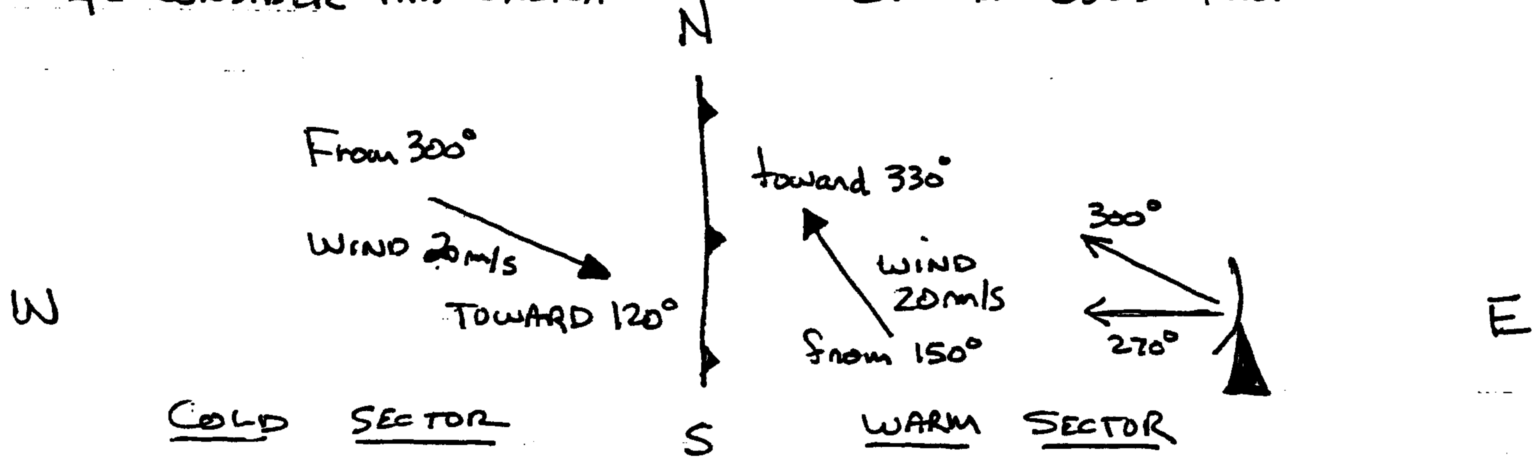
We solve 6.2 for V_r $\quad \frac{dr}{dt} = \frac{(10\,cm)(-\pi/3)}{(4\pi)(6.667\times10^{-4}\,s)} = -12.5\,m/s$

So that $-V_{r_{max}} + (-12.5\,m/s) = \underline{\underline{-50\,m/s}}$

Similarly, the answer obtained for our 3cm λ radar is found to be $-11.25\,m/s + (-3.75\,m/s) = \underline{\underline{-15\,m/s}}$

Problem 5

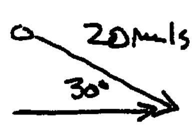
4- Consider this sketch of a cold front



Winds are horizontally and vertically uniform on both sides of front. There are abundant dust and insect scatters blowing with the wind to allow detection of the radial velocities. Our 10cm radar is operating at a PRF that gives $V_{r_{max}} > 20$ m/s

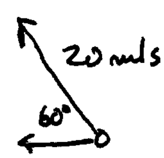
1- Calculate V_r radar would measure on each side of front when it scans front at 270° azimuth.

Cold Sector



@ 270° $\frac{V_r}{20\,m/s} = \cos 30^\circ = -17.32\,m/s$

Warm Sector



@ 270° $\frac{V_r}{20\,m/s} = \cos 60^\circ = +10.00\,m/s$

Problem 5

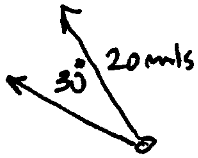
2- Calculate V_r's as above when radar scans front at 300° azimuth.

Cold Sector

Since the wind is blowing directly toward the radar

$V_R = -20.00$ m/s

Warm Sector



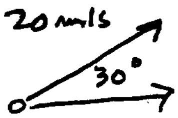
$\frac{V_r}{20 \text{ m/s}} = \cos 30^\circ = +17.32$ m/s

3- Repeat all calculations for #1 & #2, but let the wind in the warm sector blow at 20 m/s from 240° toward 60° - No change cold sector.

No change in V_rs in the cold sector.

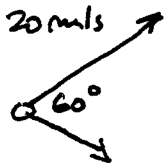
Warm Sector

When radar scans toward 270°



$V_r = -17.32$ m/s

When radar scans toward 300°



$V_r = -10.00$ m/s