

Understanding the dynamics of snowpack in Washington State - part II: complexity of snowpack

Nian She and Deniel Basketfield

Seattle Public Utilities, Seattle, Washington

Abstract. In part I of this study the strong teleconnections between large-scale atmospheric circulation patterns and the peak of snowpack measured by snow water equivalent (SWE) in Washington State were established by a linear regression model. However, the underlying dynamics of snowpack for each snow telemetry (SNOTEL) site are unknown. The statistically significant correlations between the peaks of snowpack and large-scale climate indices do not necessarily imply linear relationships among them. Moreover, these correlations do not imply a deterministic or stochastic dynamic process underlying snowpack SWE data. To better understand the complexity of snowpack dynamics in Washington State, a technique developed from nonlinear time series analysis and dynamic theory called surrogate data testing was used to investigate the complexity of snowpack dynamics in terms of determinism vs. randomness. Some relatively new concepts, such as embedding dimensions, time delay, phase re-construction, correlation dimensions and prediction errors are introduced to hydrologists and water managers to characterize the complexity and predictability of the snowpack at specified sites.

1. Introduction

Despite numerous studies that have shown the significant correlations between streamflow or snowpack and large-scale patterns of climate variation over the Pacific, little work has been done to relate this climate information to the complexity of snowpack dynamics. This may be due to oversimplified observations that snowpack is a seasonal phenomenon and its year-to-year variability can be characterized by warm years that tend to be relatively dry with light snowpack, and cool years that tend to be relatively wet with heavy snowpack. However, in part I of this study it was shown that the combined effects of climate variability and its interaction on snowpack are complex, and the peaks of snowpack as measured by snow water equivalent (SWE) at twenty-five snow telemetry (SNOTEL) sites in Washington State vary from middle December to early June from location to location. In addition, a recent study conducted by the National Assessment Synthesis Team (USGCRP, 2000) shows that the snowpack in the Columbia River basin may start to melt earlier and thus shift peak streamflow to earlier in the year, complicating reservoir flood management, and increasing the risk of late-summer shortages. It is therefore the objective of this study to identify the underlying SWE dynamics of snowpack in Washington State.

Unlike the construction of conventional statistical models or models based on the laws of physics, the SWE time series are used here to construct a topologically equivalent trajectory of a true dynamic system of snowpack SWE for each SNOTEL site under the study. This approach has solid theoretical origins dated in early1980s. Takens [1981] proved that the original dynamic system is topologically equivalent to an embedded system as long as the embedding dimension m is as twice as large as the dimension of the original system n. Here several relatively new concepts, such as embedding dimension, time delay, phase reconstruction, correlation dimensions and prediction errors will be introduced to water managers in characterizing the complexity and predictability of the snowpack at specified sites.

In the past, the complexity of snowpack was studied either by physically-based models ranging from simple energy balance constructions to full general circulation models, or by statistical models ranging from simple linear regressions to time series models. For example, Mesoscale Model version 5 (MM5) and Community Land Surface Model (CLM) developed by the National Center for Atmospheric Research was used to simulate the snowpack in the Northwestern United States (Miller, 2005), but the results are poor. In many cases the assumptions are made concerning the hydrological processes involved, and the data are thereafter inadvertently used to either calibrate or fit a prejudiced model. In this paper, a data-driven approach was taken to investigate the underlying dynamics of the snowpack. Instead of pre-assuming underlying hydrologic processes, field data were used to construct a dynamic system that is topologically equivalent to the true dynamics under the investigation. This powerful method will presents the underlying “hidden” dynamics through the data itself and without preconceived assumptions. Hence, the complexity of the data’s underlying dynamics can be characterized by the system’s dimensionality, predictability, periodicity, deterministic nonlinearity and probability. The theoretical foundation of the characterization is drawn from the Takens time-delay embedding theorem of phase space reconstruction [Takens, 1981). Takens proved that the attractor of a dynamic system in phase space can be reconstructed from a time series of a single component with the embedding vectors

$$\mathbf{x}(t_i) = \{x(t_i), x(t_i + \tau), \ldots, x(t_i + (m-1)\tau)\} \quad (1)$$

Where m denotes the embedding dimension, τ is the sample time, usually called time delay and x denotes one component of vector $\mathbf{x}$. Here two concepts are introduced: phase space and attractor. In dynamic systems, phase space is the space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the multidimensional space. The succession plot of these points presents the system's state evolving over time. A sketch of the phase portrait (its shape) may give qualitative information about the dynamics of system, such as a cycle. An attractor is a set of points to which the system evolves after a long enough time. For the set to be an attractor, trajectories that get close enough to the attractor must remain close even if slightly disturbed. Geometrically, an attractor can be a point, a curve, a manifold, or even a complicated set with fractal structures known as a strange attractor. A trajectory of the dynamical system in the attractor does not have to satisfy any special constraints except for remaining on the attractor. The trajectory may be periodic or chaotic or of any other type.

Takens embedding theorem states that if a time series comes from a dynamical system that is on an attractor, the trajectories constructed from the time series by embedding will have the same topological properties as the original one provided that $m \geq 2D+1$, where D is the dimension of the original attractor. This implies that if the original attractor has dimension D, then an embedding dimension of $m \geq 2D+1$ will be sufficient for reconstructing the attractor. In practice, choosing the length of time series, the optimal embedding dimension and time delay still remains a difficult problem, and it is critical in many applications including this one.

Once Takens embedding theorem is applied to SWE time series, the unobserved state variables may be studied in phase space, and the properties of reconstructed attractors are equivalent to that of the original system. Hence, the complexity of the original system can be characterized from the geometrical and dynamic properties of reconstructed attractors. Those properties are usually measured by dimensions. The most commonly studied dimension is called correlation dimension, which is a measure of the dimensionality of the space occupied by a set of random points. For example, if we have a set of random points on the real number line between 0 and 1, the correlation dimension will be 1, while if they are distributed on say, a triangle embedded 3D-space, the correlation dimension will be 2. This is what we would intuitively expect from a measure of dimension. The real utility of the correlation dimension is in determining the (possibly fractional) dimensions of strange attractors, and the algorithms to calculate correlation dimensions are provided by Grassberger and Procaccia [1983a, b].

However, before calculating dimensions for an attractor, necessary procedures must be undertaken with care because the algorithms, especially algorithms calculating correlation dimensions, only work under the very strict conditions that data quality and quantity are sufficient to observe clear scaling regions. Further, these algorithms may incorrectly characterize non-chaotic and even linear stochastic process as low-dimensional chaos, particularly those of power law type linear correlations [e.g. Theiler, 1986; Osborne and Provenzale, 1989]. Therefore, before building a model from the data (e.g. for prediction purposes), or applying algorithms for phase space reconstruction (e.g. calculating correlation dimensions), it is advisable to check whether the data alone suggest this type of modeling or calculation. The procedure of conducting such a test is called a surrogate data test, and will be described in the next section.

2. Surrogate data test

An important problem in hydrology is to determine whether or not an observed time series, such as streamflow, rainfall precipitation, temperature and SWE, is deterministic, contains a deterministic component, or is purely random. The surrogate data test is a statistically rigorous, foolproof framework, that can be used to test such time series data, in which surrogate data sets are generated from the original data set by preserving some structure of the original data, but destroying the deterministic structures. Then one applies some test statistics to both the surrogates and original data. The statistics chosen for the test should be robust, i.e. when the distribution of samples departs from an assumed distribution, the statistic still has test power. If the statistics in original data are different than that in the surrogates, then one may reject the null hypothesis that an underlying linear stochastic process is in effect. Otherwise, the null hypotheses cannot be rejected and a degree of underlying determinism can be assumed. The surrogate data test herein is referred as a statistical procedure that includes formulating a hypothesis, choosing a test statistic, specifying a probability of false rejection. In general, there are four standard hypotheses to test: (i) whether or not a time series is generated from a white noise; (ii) whether or not a time series is generated from a linearly filtered noise; (iii) whether or not a time series is generated from a linearly filtered noise through a static monotonic nonlinear transformation; (iv) whether or not a time series is generated from a periodic orbit with uncorrelated noise. The techniques required to conduct these tests were first suggested by Theiler, et al. [1992], extended by Schreiber and Schmitz [1996, 2000], Kantz and Schreiber [1997], Small and Tse [2002] and others.

In case (i), surrogate data sets are obtained by simply shuffling the original time series. This will destroy any temporal correlation, so that the surrogates are generated randomly but with the same probability distribution as the original time series. In case (ii) surrogate data sets are generated through the discrete Fourier transform of the original time series by shuffling the phase of complex conjugate pairs, but keeping the amplitude so that the surrogates will have the same power spectrum as the original time series, but no nonlinear determinism. In case (iii) surrogate data sets are generated as same as in case (ii) but both the power spectrum and probability distribution of the original time series are preserved. In case (iv) surrogate data sets are generated by the pseudo-periodic surrogates (PPS) algorithm developed by Small et. al. [2001, 2002], which preserve coarse deterministic features, such as periodic trends, but destroy fine structures, such as deterministic chaos. It is out of the scope of this paper to show the details of surrogate data testing, but interested readers are referred to Theiler et. al. [1992] for surrogate data testing for linear stochastic process, and Small et. al [2001, 2002] for pseudo-periodic time series and the references therein.

For the time series of SWE under this study the periodic feature is obvious. But at lower elevation sites the snowpack is influenced by regional climate, such as mild temperatures combined with the high winter-season precipitation, and therefore the snowpack fluctuations within the winter season can be large. Therefore, case (i) and (ii) are out of the question, and the question is whether or not the periodicities underlying the process are from a linear stochastic process, possibly through a static monotonic nonlinear transformation? If not, is it more complicated than an uncorrelated noisy periodic orbit? And is there any additional determinism in the system? These questions are basically hypothesis tests (iii) and (iv). So, in the rest of this section the focus is on conducting these two hypothesis tests for the SWE time series at twenty five SNOTEL sites in Washington State. For the convenience of readers, Table 1 re-list the sites studied in part I.

(Table 1)

First, it is assumed that the underlying process of snowpack is linear stochastic, possibly through a static monotonic nonlinear transformation. If the hypothesis is rejected, then the second hypothesis to be tested is that SWE time series is sampled from an uncorrelated noisy orbit. It should be noted that the rejection of a linear stochastic process only means that the underlying process is nonlinear, exhibiting only short-term stationary deterministic dynamics with uncorrelated noise, but does not mean long-term determinism or chaos is present. Therefore, two different algorithms are used to generate surrogates for these different purposes. For the first null hypothesis, the algorithm proposed by Theiler and colleagues was used to generate surrogates by shuffling the phase of the Fourier transform of the data, while preserving its power spectrum (linear correlations) and probability distribution, but destroying any additional nonlinear structures. The description of the algorithm and its applications in hydrology can be found in She and Basketfield [2005] for details. So, the PPS algorithm, which preserves coarse deterministic features, such as periodic trends, but destroys fine structures, such as deterministic chaos, will be applied in the remainder of this section.

For the second hypothesis, PPS algorithm was used to generate surrogates. The PPS data exhibit the same periodic features as the original data, but have no other deterministic structures. This was be done by reconstructing the attractor in phase space defined by (1), then iterating trajectories randomly among spatial neighbors. The algorithm is restated here from the paper by Small and Tse [2002].

Step 1. Construct the vector delay embedding $\{z_t\}_{t=1}^{N-d_w}$ from the original time series of N observations $\{y_t\}_{t=1}^{N}$ according to

$$z_t = (y_t, y_{t+\tau}, y_{t+2\tau} ,..y_{t+m\tau}) \tag{2}$$

Where the embedding dimension m and time delay τ remain to be chosen. The embedding window d_w is defined as $d_w = m\tau - 1$.

Step 2. Let $A = \{z_t \mid t = 1, 2, ,..N - d_w\}$ be the reconstructed attractor.

Step 3. An initial point $s_1 \in A$ is chosen at random, and set $i = 1$.

Step 4. Choose a near neighbor $z_j \in A$ of s_i according to the probability distribution

$$\Pr(z_j = z_t) \propto \exp\frac{-\| z_t - s_i \|}{\rho} \tag{3}$$

Where the parameter ρ is the noise radius.

Step 5. Set $s_{i+1} = z_{j+1}$, and increment i by 1.

Step 6. if i < N, then go to step 4.

Step 7. The surrogate data set $\{(s_t)_1\} = \{(s_1)_1, (s_2)_1, \ldots (s_N)_1\}$ is obtained from the first components of $\{s_t\}_t$.

The PPS algorithm is very simple, and all details can be found in Small et. al. [2001, 2002].

Figure 1 shows the original SWE time series from Pope Ridge station, along with a surrogate generated from PPS algorithm and reconstructed attractors. The original data and the surrogate appear qualitatively similar and the reconstructed attractors qualitatively look the same.

(Figure 1)

A zeroth-order nonlinear prediction error [Theiler and Prichard, 1996] was chosen as a discriminating statistic for both hypotheses tests. The reasons for choosing a zeroth-order prediction error instead of choosing other statistics, such as correlation dimensions, are that intuitively if the original data contains some deterministic structures, its prediction error on average should be smaller than that from the surrogates in which the deterministic structure were destroyed; further, it has been shown that zeroth-order nonlinear prediction error is a robust and powerful statistic even in the case when the random errors are generated from a non-Gaussian process [She and Basketfield, 2005]. The significance level for one-sided tests is set to 95 percent, so nineteen surrogate data sets are required to be generated (comparing the statistics from 19 surrogates with one in original data set gives a significant level of 95 percent). The null hypothesis may be rejected when the prediction error is smaller for the original data than for all of the nineteen surrogates.

From Table 2 one can see that the first null hypothesis was rejected at seventeen out of twenty-five sites. This indicates that the snowpack dynamics appear to be nonlinear at these sites. For the computational convenience, all zeros from non-snow cover seasons were discarded from the time series. This has no effect on the generation of surrogates and on the tests. For the remaining eight sites, the null hypothesis can not be rejected, which means that the snowpack dynamics at these sites are likely stochastic and exhibit no determinism. To further test whether or not the snowpack dynamics at these seventeen sites are uncorrelated periodic orbits, the PPS algorithm was used. One can see from Table 2 that only five sites are rejected. This indicates that the snowpack data at these five sites may not be represented by a simple uncorrelated periodic orbit, but exhibit correlated noise with a long correlation time, which may be an indication of long-term determinisms, such as chaos.

(Table 2)

When applying the PPS algorithm to the second null hypothesis, three parameters must be selected appropriately: time delay τ , embedding dimension m and the noise radius ρ . If the time delay τ is taken too small, there is almost no difference between the different components of the embedding vectors, to such an extent that all points are concentrated around the diagonal. On the other hand, if the time delay is taken too large, the different coordinates may be almost uncorrelated, such that the reconstructed attractor becomes very complicated and the original structure of the attractor is lost. The choice of time delay τ is here based on the so-called mutual information [Frazer and Swinney, 1986], which can be considered as a nonlinear analogue to linear correlation and is more adequate than an autocorrelation function when nonlinear dependencies are present. A possible rule to choose an appropriate time delay τ is to use the first minimum of the time delayed mutual information. Thus, the components of the embedding vectors can be considered independent with at least this lag. The embedding dimensions are determined using the false nearest neighbors method proposed by Kennel, et al. [1992]. For the details of how to choose the time delay and embedding dimensions, interested readers are referred to a text by Kantz and Schreiber [1997]. Similarly the noisy radius ρ must be chosen carefully. If ρ is too large then the PPS algorithm would introduce too much randomization and the periodic structures of the original data would be destroyed. Conversely, if ρ is too small then insufficient noise would produce surrogates excessively like the data, leading to great likelihood of false positive results. According to Small and Tse [2002], an intermediate ρ is that which produces surrogates that have the greatest number of short sequences identical to the data. In their study, choosing ρ in this way produces surrogates that appear qualitatively similar to the data, but lack any long term determinism. The dimensions and time delay chosen to generate surrogates were also used by zeroth-order prediction error in the tests.

3. Characterization of snowpack

The spatial and temporal variation of snowpack is influenced by many factors such as local topography, temperature, wind speed, solar radiation, air and soil moisture, and consequent microclimates. In part I of this study, it was shown that the effects of large-scale climate indices and the interactions between them on the snowpack in Washington State are significant. Figure 2 shows an example of snowpack measured by SWE at Cougar Mountain of Washington State from 1981 to 1992. It can be seen that the magnitude of snowpack varies not only year by year, but within years. Multiple peaks are evident within the same snow cover season from the plot. This many be due to the typical regional weather pattern of warm temperature and high precipitation during the winter months. From Table 2 one can see that the hydrologic responses of the snowpack at Cougar Mountain to those factors are characterized by embedding dimension ($m = 4$), time delay ($\tau = 72$, corresponding to autocorrelation), and linearity (a linear stochastic process cannot be rejected). This indicates that at this low-elevation site the randomness of the weather pattern dominates the underlying hydrologic process involved. It should be noted that the nonlinear components in a system do not guarantee the nonlinear behavior of the system because the random noise may be so strong that the nonlinearity is destroyed by the noise.

Among the twenty-five sites under the study seventeen of them appeared to have some kind of nonlinear structures and five of them appeared to have long-term determinism. One should keep in mind that a rejection of linear stochastic process in hypothesis 1 only indicates nonlinearity, not necessarily low-dimensional deterministic chaos. Only when both hypotheses are rejected, the data may be analyzed further to detect possible low-dimensional deterministic. Figure 3 shows the local slopes of correlation sum calculated for the daily SWE at Pope Ridge SNOTEL site. Note that both hypotheses were rejected at this site. The embedding is done with a time delay of 70 days in $m = 1, 2, \ldots 10$ dimensions. It can be seen clearly that for ε between 7 and 12 a scaling region was found. For a self-similar object the local scaling exponent is convergent to a constant for all embedding dimensions greater than a threshold. In this case, the local scaling exponent converges to a neighborhood of 2 for all embedding dimensions larger than 4, where the plateau is convincing. So, this scaling exponent can be used to estimate the correlation dimension of the attractor reconstructed from daily SWE at Pope Ridge SNOTEL site. This correlation dimension is a fraction slightly large than 2. A fraction correlation dimension implies long-term determinism or chaos in a system.

By characterizing the snowpack in these twenty-five SNOTEL sites, it was found that the underlying hydrologic processes for snowpack vary from linear stochastic, to nonlinear to chaos. Therefore, caution must be exercised when studying the long-range effects of climate change on snowpack. For a chaotic system, even the governing equations and parameters are correct, but a slight difference in initial conditions will result a large error for long-term simulations. This is called sensitivity to initial conditions, which is a signature of chaos.

4. Conclusion and discussion

To date, the characterization of precipitation/snowpack and streamflow has largely been done by either applying physically-based models or traditional statistical analysis. In either case, it pre-assumes an underlying hydrologic process or probability distribution of the data, and as a result the field data are used to either calibrate or fit a prejudiced model. In this paper, a new method called a surrogate data test was used to characterize the snowpack without pre-assumptions. It was applied to observed time series of SWE to detect significant nonlinearity and determinism.

For the past quarter of a century, the techniques used in surrogate data testing, such as embedding dimensions, time delay, phase reconstruction, correlation dimensions and prediction errors have rarely been applied to hydrology, hence, they are fairly new and recent to water resource managers. For this reason, this study demonstrated that the complexity of the processes underlying snowpack dynamics can be characterized by the dimensions of a reconstructed attractor in phase space, i.e., linearity vs. nonlinearity and determinism vs. uncorrelated noisy periodicities. For example, embedding dimension m gives an upper bound of the dimensions of the original dynamic system. Moreover, this bound is determined from measured data rather than through a pre-assumed and potentially prejudiced model subjectively selected by researchers.

These techniques together with the surrogate data test were applied to characterize snowpack at twenty-five SNOTEL sites in Washington State. As has been shown in the last section, the complexity of these SWE are characterized by the length of time delays, the degrees of embedding dimensions, linear stochastic vs. nonlinear dynamic characteristics, and deterministic vs. uncorrelated noisy periodic processes presented in the snowpack.

The dynamics underlying the snowpack are complex hydrologic processes. Even the knowledge that certain components of this hydrologic system may exhibit nonlinear behavior does not necessarily lead to the conclusion that a specific output of the system (e.g. SWE) is consequently nonlinear, or that this nonlinearity will be proven evident in snowpack dynamics. Instead of pre-supposing the underlying dynamics, a surrogate data test was used to represent the dynamics in an “inverse” manner. It has been shown in section 2 that the surrogate data test is a powerful statistical tool that is capable of distinguishing nonlinear and chaotic dynamics from linear stochastic and uncorrelated noisy periodic processes.

In conclusion, the underlying hydrologic processes of snowpack at the twenty-five SNOTEL sites in Washington State exhibit three different behaviors: linear stochastic, nonlinear but short-term stationary deterministic, and possible chaotic processes. Understanding these dynamics will help researchers to adequately model the systems and make reliable predictions.

Acknowledgements. The author would like to thank Small for his computer program for PPS algorithm and suggestions in implementation of the algorithm.

References

Frazer, A.M. and H. L. Swinney, Independent coordinates in strange attractors from mutual information, Phys. Rev. A, 33, 1134-1140, 1986.

Grassberger P. and I. Procaccia, Characterization of strange attractors, Phys. Rev Lett., 50, 346-349, 1983a.

Grassberger, P. and I. Procaccia, Estimation of the Kolmogorov entropy from a chaotic signal, Phys. Rev. A, 28, 2591, 1983b.

Kantz, H. and T. Schreiber, Nonlinear Time Series Analysis, Cambridge University Press, Cambridge, 1997.

Kennel, M. B., R. Brown, and H. D. I. Abarbanel, Determining embedding dimension for phase-space reconstruction using a geometrical construction, Phys. Rev. A, 45, 3403, 1992.

Miller, N., Recent advances in regional climate system modeling and climate change analyses of extreme heat, Lawrence Berkeley National Laboratory, for the California Energy Commission, PIER Energy-Related Environmental Research, CEC-500-2005-016, 2005.

Osborne, A. R.and A. Provenzale, Finite correlation dimension for stochastic systems with power-law spectra, Physica D, 35, 357-381, 1989.

Schreiber, T. and A. Schmitz, Improved surrogate data for nonlinearity tests, Phys. Rev. Lett., 77, 635-638, 1996.

Schreiber, T. and A. Schmitz, Surrogate time series, Physica D, 142, 346-382, 2000.

She, N. and D. Basketfield, Streamflow dynamics at the Puget Sound, Washington: Application of a surrogate data method, Nonlinear Processes in Geophysics, 12, 461–469, 2005.

Small, M. and C. K. Tse, Applying the method of surrogate data to cyclic time series, Physica D 164, 187-201, 2002.

Small, M., D. J. Yu, and R. G. Harrison, A surrogate test for pseudo-periodic time series data, Phys. Rev Lett.**,** 87, 188101, 2001.

Takens, F., Detecting strange attractors in turbulence, in: Lecture Notes in Mathematics, edited by Young, L.S., Springer, Berlin, 1981.

Theiler, J., S. Eubank, A. Longtin, B. Galdrikian, and J. D. Farmer, Testing for nonlinearity in time series: the method of surrogate data, Physica D, 58, 77-94, 1992.

Theiler, J. and D. Prichard, Using ‘Surrogate Surrogate Data’ to calibrate the actual rate of false positives in tests for nonlinearity in time series, Fields Institute Communications, 11, 99, 1996.

Table 1. Washington State Snotel Sites

Name	Time Period	Elevation (m)	Name	Time Period	Elevation (m)
Blewett Pass	82-05	1301	Park Creek Ridge	79-05	1402
Bumping Ridge	84-05	1402	Pigtail Peak	82-05	1798
Bunchgrass	84-05	1524	Pope Ridge	82-05	1079
Corral Pass	82-05	1829	Potato Hill	84-05	1372
Cougar Mountair	79-05	975	Rainy Pass	83-05	1457
Fish Lake	84-05	1027	Salmon Meadow	84-05	1372
Green Lake	83-05	1829	Sheep Canyon	84-05	1228
Grouse Camp	83-05	1640	Stampade Pass	83-05	1177
Harts Pass	83-05	1981	Stevens Pass	81-05	1241
Lone Pine	82-05	1158	Surprise Laks	80-05	1295
Lyman Lake	84-05	1798	Upper Wheeler	82-05	1341
Morse Lake	84-05	1646	White Pass	81-05	1372
Paradise	84-05	1561			

Table 2. Surrogate Data Test of Hypotheses 1 and 2 for Twenty-five SNOTEL Sites in Washington State

Name	Period	Snow Cover Season	Embedding Dimensions	Time Delay(days)	First Hypothesis	Second Hypothesis
Blewett Pass	1982-2005	Oct-May	5	71	Not reject	Not tested
Bumping Ridge	1984-2005	Oct - Jun	5	79	Reject	Reject
Bunchgrass	1984-2005	Oct - Jun	4	71	Reject	Not reject
Corral Pass	1982-2005	Oct - Jun	4	82	Reject	Reject
Cougar Mountain	1982-2005	Oct-May	4	72	Not reject	Not tested
Fish Lake	1984-2005	Oct - Jun	4	73	Reject	Not reject
Green Lake	1983-2005	Oct - Jun	4	75	Reject	Reject
Grouse Camp	1983-2005	Oct - Jun	7	76	Reject	Reject
Harts Pass	1983-2005	Oct - Jun	4	94	Reject	Not reject
Lone Pine	1982-2005	Oct - Jun	5	71	Not reject	Not tested
Lyman Lake	1984-2005	Oct-Jul	5	77	Reject	Not reject
Morse Lake	1984-2005	Oct - Jun	4	75	Reject	Not reject
Paradise	1984-2005	Oct-Jul	4	89	Reject	Not reject
Park Creek Ridge	1979-2005	Oct - Jun	4	73	Reject	Not reject
Pigtail Peak	1982-2005	Oct-Jul	5	83	Reject	Not reject
Pope Ridge	1982-2005	Oct-May	5	70	Reject	Reject
Potato Hill	1984-2005	Oct - Jun	5	73	Not reject	Not tested
Rainy Pass	1983-2005	Oct - Jun	4	79	Reject	Not reject
Salmon Meadow	1984-2005	Oct-May	6	65	Reject	Not reject
Sheep Canyon	1984-2005	Oct - Jun	5	81	Not reject	Not tested
Stampade Pass	1983-2005	Oct - Jun	4	71	Not reject	Not tested
Stevens Pass	1981-2005	Oct - Jun	5	76	Reject	Not reject
Surprise Laks	1980-2005	Oct - Jun	3	81	Not reject	Not tested
Upper Wheeler	1982-2005	Oct-May	5	64	Reject	Not reject
White Pass	1981-2005	Oct-June	5	71	Not reject	Not tested

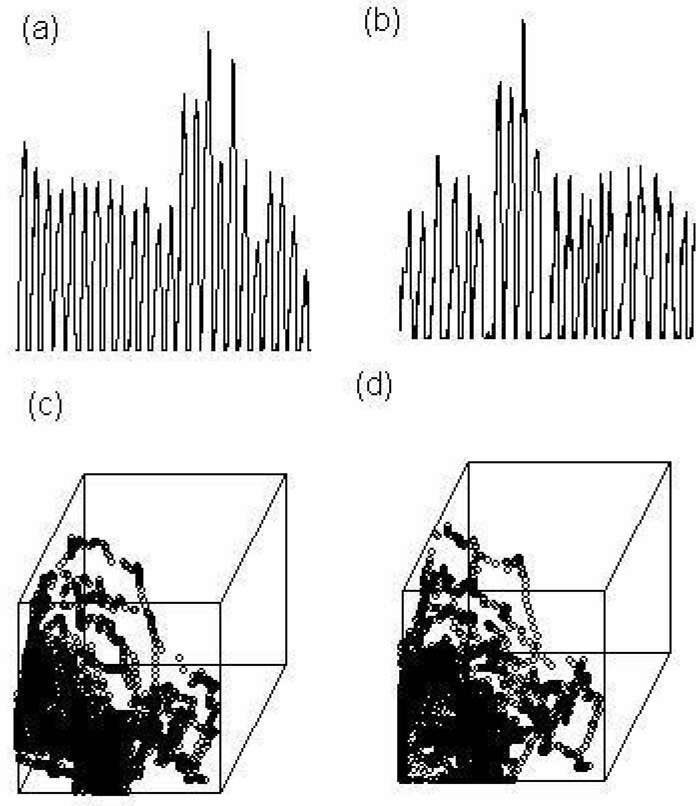

Figure 1. The top panels show the time series plots for (a) the original SWE, and (b) the PPS data. The bottom panels show the Reconstructed attractors (time delay $\tau = 70$) for (c) the original SWE, and (d) PPS data.

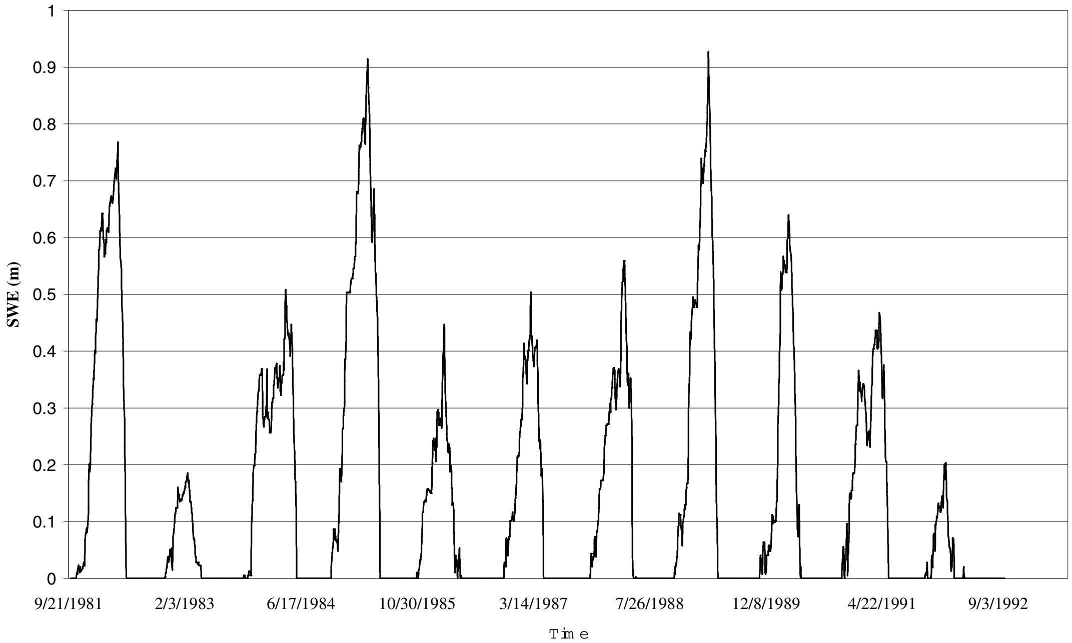

Figure 2. Snow Water Equivalent (SWE) Measured at Cougar Mountain Washington State from 1981 to 1992

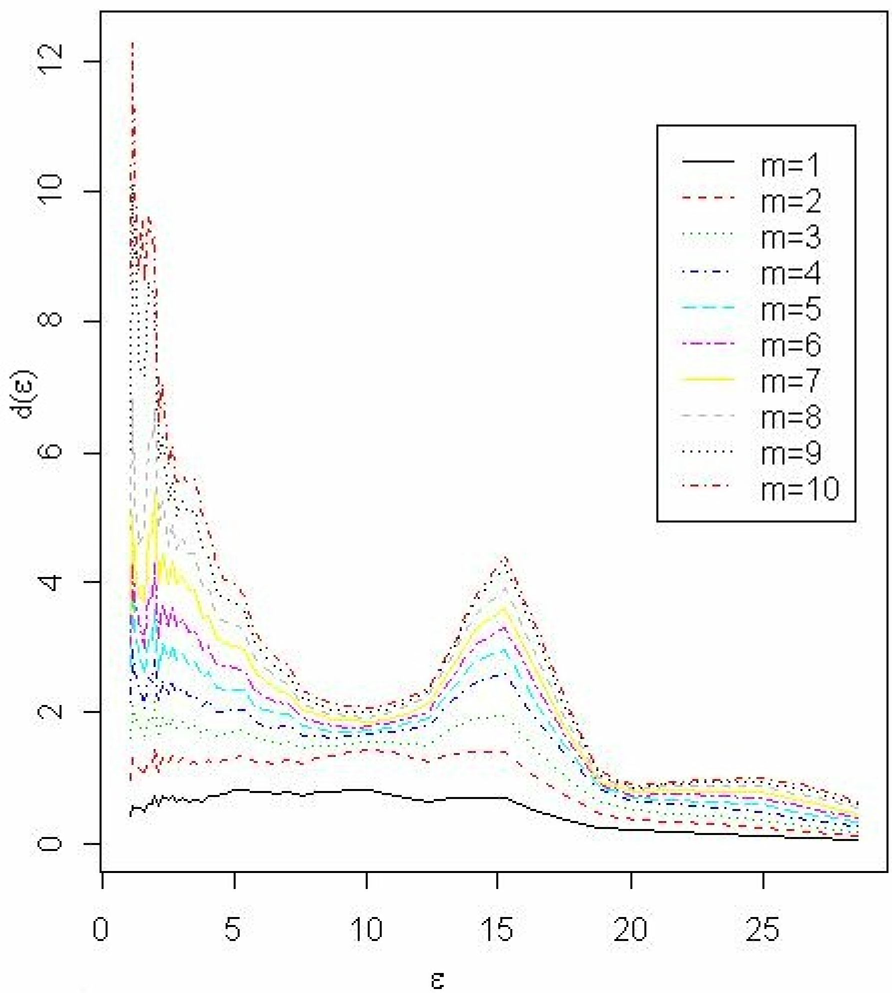

Figure 3. The local slopes d(ε) of the correlation sum for daily SWE at Pope Ridge SNOTEL sites. Embedding is done with a delay of 70 days in m=1, 2, ..., 10 dimensions.