Uncertainty Analysis of the Risk of Failure for Generalized Logistic Distribution

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Abstract

The concepts of risk and uncertainty are commonly utilized for designing and evaluating hydraulic structures such as spillways and dikes. In this study, uncertainty analysis is performed for the risk of failure based on the generalized logistic distribution that is recently recommended as an appropriate probability distribution for flood frequency analysis in UK by Institute of Hydrology. For this purpose, we derived the expected values and the variances of the risk of failure based on the methods of moments, maximum likelihood, and probability weighted moments for the generalized logistic model. The derived variances of the risk of failure based on the methods of moments, maximum likelihood, and probability weighted moments are functions of sample size, design life, and non-exceedance probability. In addition, simulation experiments have been performed to figure out the behavior of the risk of failure for various the sample sizes, design lives, non-exceedance probabilities, and coefficients of variation.

Keywords : Uncertainty, risk of failure, generalized logistic distribution, simulation experiments

Introduction

Statistical concepts and methods are routinely utilized in a number of design and management problems in engineering hydrology. This is because most if not all hydrological processes have some degree of randomness and uncertainty. Thus concepts of risk and certainty are commonly utilized for designing and evaluating hydraulic structures such as spillways and dikes. The random occurrence of annual floods is generally considered using flood frequency analysis where non-parametric and parametric methods and models are routinely applied (Stedinger et al., 1993).

Probability models such as the lognormal, gamma, log-Pearson type III, and general extreme value (GEV) have been widely used for fitting the frequency distribution of flood data. The sample data may suggest one or more candidate models which may be considered for the data at hand. While fitting a particular model has become relatively a simple task, the difficulty lies in selecting an appropriate model to be used for making design or management decisions. In many countries and regions of the world, guidelines and manuals have been developed to suggesting a particular distribution for a certain type of hydrologic data. For example, Flood Estimation Handbook (Institute of Hydrology, 1999) is a manual that suggests the generalized logistic distribution for flood frequency analysis in the UK. However, finding an appropriate design distribution is still controversial problem in actual engineering practice. Because the available flood sample is of limited size, the estimated parameters and consequently the flood quantiles are uncertain quantities. Estimating those uncertainties has been of much interest in literature for quite some time (NERC, 1975; Kite 1988; Chowdhury and Stedinger 1991). Because of the same reason, in the inverse estimation problem, i.e. in estimating the return period of a known flood magnitude and consequently the risk of failure, one must consider the associated uncertainties. While quantifying the uncertainty of flood quantiles has been extensively studied in literature, however, this is not the case with the uncertainty of the risk of failure. The purpose of this paper is to propose a procedure for quantifying the uncertainty of the hydrologic risk (or reliability) of hydraulic structures based on the generalized logistic model.

Return Period and Risk of Failure

Once a probability model is specified and its parameters are estimated from the available data, one can determine a flood quantile for any non-exceedance probability. For example, assuming that the distribution of annual maximum floods is represented by $F(x)$ one may determine the flood value x corresponding to a given non-exeedance probability q such that $F(x) = q$ and $x_q = F^{-1}(q)$ is the flood quantile. Also if flood events are independent and the exceedance probability p of the design flood remains constant over the years, the return period T is determined as $T = 1/p$. Thus such flood quantile is commonly denoted as x_T and is called the T-year flood. In flood engineering practice, the return period has been defined as the average number of years to the first occurrence of a flood event of magnitude greater than a predefined design flood (Kite, 1988). In the context of designing hydraulic structures such as drainage systems, spillways, etc. generally the return period T is specified according to the type of structure to be designed and the design flood is determined from the frequency distribution of the corresponding flood data.

Return period and risk of failure are related quantities. We will define failure as that situation in which a flood exceeding the design flood occurs. The hydrologic risk of failure of a flood related hydraulic structure is typically defined as the probability that the number of floods greater than the design flood in an n-year period is greater or equal to one. Assuming that the annual floods are independent and identically distributed, it may be shown that the risk of failure is a direct function of the return period T of the corresponding design flood. It is given by (Yen 1970; Chow et al., 1988)

$$R = 1 - (1-p)^n = 1 - q^n = 1 - (1 - 1/T)^n \tag{1}$$

where, R is the hydrologic risk of failure and n is the design life.

Uncertainty of the Risk of Failure

The generalized logistic (GL) distribution is a generalization of the 2-parameter logistic distribution and is also a special case of the kappa distribution. This generalization of the logistic distribution differs from other distributions defined in the literature. It is a reparameterized version of the log-logistic distribution presented by Ahmad et al. (1988). The name reflects the distribution's similarity to the generalized Pareto and generalized extreme value distributions (Hosking and Wallis, 1997). The cumulative distribution function of the GL distribution are defined respectively as

$$F(x) = \left[1 + \left\{1 - \frac{\beta}{\alpha}(x - x_0)\right\}^{1/\beta}\right]^{-1} \tag{2}$$

where, x_0 is the location parameter, α is the scale parameter, and β is the shape parameter. The range of possible values for the GL distribution is given by

$$x_0 + \alpha / \beta \le x < \infty \quad \text{for} \quad \beta < 0 \tag{3}$$

$$-\infty < x \le x_0 + \alpha / \beta \quad \text{for} \quad \beta > 0 \tag{4}$$

The parameters may be estimated from the sample $X_1, ..., X_N$ where N is the sample size. Let $\hat{x}_0$, $\hat{\alpha}$, and $\hat{\beta}$ denote the estimators of x_0, α, and β respectively. Thus, for a given design flood peak x_q, one can get the corresponding non-exceedance probability q from Eq. (1). Since q depends on the unknown parameters, the estimator of q is given by

$$\hat{q} = \left[1 + \left\{ 1 - \hat{\beta} \left(\frac{x - \hat{x}_0}{\hat{\alpha}} \right) \right\}^{1/\hat{\beta}} \right]^{-1} \tag{5}$$

Likewise, from Eq. (1) the estimator of the corresponding risk of failure $\hat{R}$ in an n-year period is given by

$$\hat{R} = 1 - (1 - \hat{p})^n = 1 - \hat{q}^n \tag{6}$$

Our concern is in assessing the uncertainty of this estimator of risk of failure.
From Eq. (5) the first order approximation of the expected value of $\hat{R}$ is given by

$$E(\hat{R}) \approx 1 - \left[E(\hat{q}) \right]^n \tag{7}$$

Also from Eq. (5) the term $E(\hat{q})$ in (7) may be approximated by

$$E(\hat{q}) = \left[1 + \left\{ 1 - E(\hat{\beta}) \left(\frac{x - E(\hat{x}_0)}{E(\hat{\alpha})} \right) \right\}^{1/E(\hat{\beta})} \right]^{-1} \tag{8}$$

If $\hat{x}_0$, $\hat{\alpha}$, and $\hat{\beta}$ are unbiased, then $E(\hat{q}) \approx q$ and consequently Eq. (7) may be written as

$$E(\hat{R}) \approx 1 - q^n = 1 - \left[1 + \left\{ 1 - \hat{\beta} \left(\frac{x - \hat{x}_0}{\hat{\alpha}} \right) \right\}^{1/\hat{\beta}} \right]^{-n} \tag{9}$$

Likewise, the first order approximation of the variance of $\hat{R}$ can be derived as

$$
\begin{aligned}
Var(\hat{R}) &\approx \left(\frac{\partial \hat{R}}{\partial \hat{x}_0}\right)^2 Var(\hat{x}_0) + \left(\frac{\partial \hat{R}}{\partial \hat{\alpha}}\right)^2 Var(\hat{\alpha}) + \left(\frac{\partial \hat{R}}{\partial \hat{\beta}}\right)^2 Var(\hat{\beta}) \\
&+2\left(\frac{\partial \hat{R}}{\partial \hat{x}_0}\right)\left(\frac{\partial \hat{R}}{\partial \hat{\alpha}}\right) Cov(\hat{x}_0,\hat{\alpha}) + 2\left(\frac{\partial \hat{R}}{\partial \hat{x}_0}\right)\left(\frac{\partial \hat{R}}{\partial \hat{\beta}}\right) Cov(\hat{x}_0,\hat{\beta}) + 2\left(\frac{\partial \hat{R}}{\partial \hat{\alpha}}\right)\left(\frac{\partial \hat{R}}{\partial \hat{\beta}}\right) Cov(\hat{\alpha},\hat{\beta})
\end{aligned} \tag{10}
$$

(1) Method of Moments (MOM)

By using the first three sample moments, the variance of $\hat{R}$ can be written

$$
\begin{aligned}
Var(R) = &\left(\frac{\partial R}{\partial m_1^{'}}\right)^2 Var(m_1^{'}) + \left(\frac{\partial R}{\partial m_2}\right)^2 Var(m_2) + \left(\frac{\partial R}{\partial m_3}\right)^2 Var(m_3) \\
&+2\left(\frac{\partial R}{\partial m_1^{'}}\right)\left(\frac{\partial R}{\partial m_2}\right) Cov(m_1^{'},m_2) + 2\left(\frac{\partial R}{\partial m_1^{'}}\right)\left(\frac{\partial R}{\partial m_3}\right) Cov(m_1^{'},m_3) \\
&+2\left(\frac{\partial R}{\partial m_2}\right)\left(\frac{\partial R}{\partial m_3}\right) Cov(m_2,m_3)
\end{aligned} \tag{11}
$$

The derivatives in Eq. (11) may be obtained as

$$\frac{\partial R}{\partial m_1^{'}} = -nF^{n-1}(x)\frac{\partial F(x)}{\partial m_1^{'}} = -\frac{n}{\alpha} q^{n+1}(q^{-1}-1)^{1-\beta} \tag{12}$$

$$\frac{\partial R}{\partial m_2} = -nF^{n-1}(x)\frac{\partial F(x)}{\partial m_2} = -\frac{n}{\alpha} q^{n+1}(q^{-1}-1)^{1-\beta} \frac{1}{2\sqrt{\mu_2}}\left\{K_T - 3\gamma_1 \frac{\partial K_T}{\partial \gamma_1}\right\} \tag{13}$$

$$\frac{\partial R}{\partial m_3} = -nF^{n-1}(x)\frac{\partial F(x)}{\partial m_3} = -\frac{n}{\alpha} q^{n+1}(q^{-1}-1)^{1-\beta} \frac{1}{\mu_2}\frac{\partial K_T}{\partial \gamma_1} \tag{14}$$

where, the frequency factor $\hat{K}_T$ is (Chow, 1951)

$$\hat{K}_T = \frac{\hat{\beta}}{\left|\hat{\beta}\right|} \frac{\Gamma(1+\hat{\beta})\Gamma(1-\hat{\beta}) - (T-1)^{-\hat{\beta}}}{\left\{\Gamma(1+2\hat{\beta})\Gamma(1-2\hat{\beta}) - \Gamma^2(1+\hat{\beta})\Gamma^2(1-\hat{\beta})\right\}^{1/2}} \tag{15}$$

And the variance and covariances of the moments are given by Kendall and Stewart (1963) as in Eqs. (16) to (21).

$$Var(m_1^{'}) = \frac{1}{n}\mu_2 \tag{16}$$

$$Var(m_2) = \frac{1}{n}(\mu_4 - \mu_2^2) \tag{17}$$

$$Var(m_3) = \frac{1}{n}(\mu_6 - \mu_3^2 - 6\mu_4\mu_2 + 9\mu_2^3) \tag{18}$$

$$Cov(m_1^{'}, m_2) = \frac{1}{n}\mu_3 \tag{19}$$

$$Cov(m_1^{'}, m_3) = \frac{1}{n}(\mu_4 - 3\mu_2^2) \tag{20}$$

$$Cov(m_2, m_3) = \frac{1}{n}(\mu_5 - 4\mu_{\backslash 3}\mu_2) \tag{21}$$

Then, after appropriate substitutions and simplifications the variances of $\hat{R}$ are obtained for the method of moments as shown below

$$\begin{aligned} &Var(R) \\ &= \frac{n\mu_2}{\alpha^2} q^{2(n+1)} (q^{-1}-1)^{2(1-\beta)} + \frac{n(\mu_4 - \mu_2^2)}{4\mu_2\alpha^2} q^{2(n+1)} (q^{-1}-1)^{2(1-\beta)} \ (K_T - 3\gamma_1 \frac{\partial K_T}{\partial \gamma_1}) \\ &+ \frac{n}{\alpha^2\mu_2^2} q^{2(n+1)} (q^{-1}-1)^{2(1-\beta)} \ (\frac{\partial K_T}{\partial \gamma_1})^2 \ (\mu_6 - \mu_3^2 - 6\mu_4\mu_2 + 9\mu_2^3) \\ &+ \frac{n\mu_3}{\alpha^2\sqrt{\mu_2}} q^{2(n+1)} (q^{-1}-1)^{2(1-\beta)} \ (K_T - 3\gamma_1 \frac{\partial K_T}{\partial \gamma_1}) \\ &+ 2\frac{n}{\alpha^2\mu_2} q^{2(n+1)} (q^{-1}-1)^{2(1-\beta)} \frac{\partial K_T}{\partial \gamma_1} \ (\mu_4 - 3\mu_2^2) \\ &+ \frac{n}{\alpha^2\mu_2^{3/2}} q^{2(n+1)} (q^{-1}-1)^{2(1-\beta)} \ (K_T - 3\gamma_1 \frac{\partial K_T}{\partial \gamma_1}) \ \frac{\partial K_T}{\partial \gamma_1} \ (\mu_5 - 4\mu_3\mu_2) \end{aligned} \tag{22}$$

Substituting $q = (1+e^{-y})^{-1}$ into Eq. (22) yields

$$Var(R) = \frac{n}{\alpha^2} \frac{\mu_2 e^{-2y(1-\beta)}}{(1+e^{-y})^{2(n+1)}} \left[\begin{aligned} &1 + \frac{1}{4}(K_T - 3\gamma_1 \frac{\partial K_T}{\partial \gamma_1}) \ (\gamma_2 - 1) + (\frac{\partial K_T}{\partial \gamma_1})^2 (\gamma_4 - \gamma_1^2 - 6\gamma_2 + 9) \\ &+ (K_T - 3\gamma_1 \frac{\partial K_T}{\partial \gamma_1})\gamma_1 + 2\frac{\partial K_T}{\partial \gamma_1}(\gamma_2 - 3) \\ &+ \left\{ K_T \frac{\partial K_T}{\partial \gamma_1} - 3\gamma_1 (\frac{\partial K_T}{\partial \gamma_1})^2 \right\} (\gamma_3 - 4\gamma_1) \end{aligned} \right] \tag{23}$$

(2) Method of Maximum Likelihood (ML)

First, define y as

$$y = -\frac{1}{\beta}\log\left\{1 - \beta\left(\frac{x - x_0}{\alpha}\right)\right\} \tag{24}$$

Substituting Eq. (24) into Eq. (6), then

$$R = 1 - \left(1 + e^{-y}\right)^{-n} \tag{25}$$

The partial derivatives of y are given by Eqs. (26) to (28)

$$\frac{\partial y}{\partial x_0} = -\frac{1}{\alpha} e^{\beta y} \tag{26}$$

$$\frac{\partial y}{\partial \alpha} = -\frac{1}{\alpha\beta}(e^{\beta y} - 1) \tag{27}$$

$$\frac{\partial y}{\partial \beta} = -\frac{y}{\beta} + \frac{1}{\beta^2}(e^{\beta y} - 1) \tag{28}$$

The partial derivatives of R with respect to x_0, α, and β are

$$\frac{\partial R}{\partial x_0} = \frac{n}{\alpha}(q^{-1} - 1)^{1-\beta} q^{n+1} \tag{29}$$

$$\frac{\partial R}{\partial \alpha} = \frac{n}{\alpha\beta}(q^{-1} - 1)\left\{(q^{-1} - 1)^{-\beta} - 1\right\} q^{n+1} \tag{30}$$

$$\frac{\partial R}{\partial \beta} = \frac{n}{\beta^2}(q^{-1} - 1)\left\{-(q^{-1} - 1)^{-\beta} + 1 - \beta \log(q^{-1} - 1)\right\} q^{n+1} \tag{31}$$

And the variance and covariances of parameter of the GL distribution are given by Shin et al. (2006) as in Eqs. (32) to (37).

$$Var(x_0) = \frac{3\alpha^2}{D}(-S_1S_3 + S_4\beta^2 + S_1S_3S_4\beta^2 - S_5^{\,2} - 1) \tag{32}$$

$$Var(\alpha) = \frac{3\alpha^2\beta^2}{D}(S_1^{\,2}S_3g_2 - 2S_1S_5g_2 + S_1S_4g_2\beta^2 - S_5^{\,2} + 2S_1S_2S_5 - S_1^{\,2}S_2^{\,2}) \tag{33}$$

$$Var(\beta) = \frac{3\beta^4}{D} S_1(g_2 + S_1S_3g_2 - S_1S_2^{\,2}) \tag{34}$$

$$Cov(x_0, \alpha) = -\frac{3\alpha^2\beta}{D}(-S_1S_2S_5 + S_1S_2S_4\beta^2 - S_5^{\,2} + S_1S_3S_5 + S_5 - S_1S_2) \tag{35}$$

$$Cov(x_0, \beta) = -\frac{3\alpha\beta^2}{D} S_5(-S_1S_2 + 1 + S_1S_3) \tag{36}$$

$$Cov(\alpha, \beta) = \frac{3\alpha\beta^3}{D} S_1(-S_5g_2 + S_1S_3g_2 + g_2 + S_2S_5 - S_1S_2^{\,2}) \tag{37}$$

where, $S_1 = 1 - \beta^2$, $S_2 = g_2 - g_1$, $S_3 = g_2 - 2g_1$, $S_4 = 1 + \dfrac{1}{\beta^2} + \dfrac{\pi^2}{3}$,

$S_5 = g_1\left\{1 - \dfrac{\beta(1-\beta^2)}{\psi(1-\beta) - \psi(\beta)}\right\}$, $g_r = \Gamma(1 + r\beta)\Gamma(1 - r\beta)$, and

$$D = N\{(S_2{}^2 - S_3 g_2)S_1{}^2 + (g_2 + S_1 S_3 g_2 - S_1 S_2{}^2)S_1 S_4 \beta^2 + (2S_2 - g_2 - \frac{1}{S_1} - S_3)S_1 S_5{}^2 - S_1 g_2\}$$

.

Finally, substituting Eqs. (29) into (31) and the variance and covariance terms into Eq. (10) yields

$$\begin{aligned}
Var(R) &= \frac{n^2}{\alpha^2}\left(q^{-1}-1\right)^{2(1-\beta)} q^{2(n+1)} Var(x_0) \\
&+ \frac{n^2}{\alpha^2\beta^2}\left(q^{-1}-1\right)^2 \left\{(q^{-1}-1)^{-\beta}-1\right\}^2 q^{2(n+1)} Var(\alpha) \\
&+ \frac{n^2}{\beta^4}\left(q^{-1}-1\right)^2 \left\{-(q^{-1}-1)^{-\beta}+1-\beta\log(q^{-1}-1)\right\}^2 q^{2(n+1)} Var(\beta) \\
&+ 2\frac{n^2}{\alpha^2\beta}\left(q^{-1}-1\right)^{2-\beta} \left\{(q^{-1}-1)^{-\beta}-1\right\} q^{2(n+1)} Cov(x_0,\alpha) \\
&+ 2\frac{n^2}{\alpha\beta^2}\left(q^{-1}-1\right)^{2-\beta} \left\{-(q^{-1}-1)^{-\beta}+1-\beta\log(q^{-1}-1)\right\} q^{2(n+1)} Cov(x_0,\beta) \\
&+ 2\frac{n^2}{\alpha\beta^3}\left(q^{-1}-1\right)^2 \left\{(q^{-1}-1)^{-\beta}-1\right\}\left\{-(q^{-1}-1)^{-\beta}+1-\beta\log(q^{-1}-1)\right\} q^{2(n+1)} Cov(\alpha,\beta)
\end{aligned} \tag{38}$$

(3) Method of Probability Weighted Moments (PWM)

The partial derivatives of R with respect to x_0, α, and β are Eqs. (29) to (31) and the variance and covariances of PWM parameter estimator for the GL distribution are given by Shin et al. (2006) as in Eqs. (32) to (37).

$$Var(\hat{x}_0) = N^{-1}\left[g_{00}{}^2 V_{00} + g_{01}{}^2 V_{11} + g_{02}{}^2 V_{22} + 2g_{00}g_{01}V_{01} + 2g_{00}g_{02}V_{02} + 2g_{01}g_{02}V_{12}\right] \tag{39}$$

$$Var(\hat{\alpha}) = N^{-1}\left[g_{10}{}^2 V_{00} + g_{11}{}^2 V_{11} + g_{12}{}^2 V_{22} + 2g_{10}g_{11}V_{01} + 2g_{10}g_{12}V_{02} + 2g_{11}g_{12}V_{12}\right] \tag{40}$$

$$Var(\hat{\beta}) = N^{-1}\left[g_{20}{}^2 V_{00} + g_{21}{}^2 V_{11} + g_{22}{}^2 V_{22} + 2g_{20}g_{21}V_{01} + 2g_{20}g_{22}V_{02} + 2g_{21}g_{22}V_{12}\right] \tag{41}$$

$$\begin{aligned}
Cov(\hat{x}_0,\hat{\alpha}) = N^{-1}\big[& g_{10}g_{00}V_{00} + g_{11}g_{01}V_{11} + g_{12}g_{02}V_{22} \\
&+ (g_{10}g_{01} + g_{11}g_{00})V_{01} + (g_{10}g_{02} + g_{12}g_{00})V_{02} + (g_{11}g_{02} + g_{12}g_{01})V_{12}\big]
\end{aligned} \tag{42}$$

$$\begin{aligned}
Cov(\hat{x}_0,\hat{\beta}) = N^{-1}\big[& g_{20}g_{00}V_{00} + g_{21}g_{01}V_{11} + g_{22}g_{02}V_{22} \\
&+ (g_{20}g_{01} + g_{21}g_{00})V_{01} + (g_{20}g_{02} + g_{22}g_{00})V_{02} + (g_{21}g_{02} + g_{22}g_{01})V_{12}\big]
\end{aligned} \tag{43}$$

$$Cov(\hat{\alpha},\hat{\beta}) = N^{-1}\left[g_{20}g_{10}V_{00} + g_{21}g_{11}V_{11} + g_{22}g_{12}V_{22} + (g_{20}g_{11} + g_{21}g_{10})V_{01} + (g_{20}g_{12} + g_{22}g_{10})V_{02} + (g_{21}g_{12} + g_{22}g_{11})V_{12}\right] \quad (44)$$

where, $d_1 = \psi(1+\beta) - \psi(1-\beta)$, $g_1 = \Gamma(1+\beta)\Gamma(1-\beta)$

$$g_{00} = \frac{1}{g_1}\left\{\frac{(1-\beta)^2(g_1-1)}{\beta^2} + 1 - \frac{1-\beta}{\beta}d_1\right\},\; g_{01} = \frac{1}{g_1}\left\{\frac{2(2\beta-3)(g_1-1)}{\beta^2} + \frac{2(3-\beta)}{\beta}d_1\right\},$$

$$g_{02} = \frac{1}{g_1}\left\{\frac{6(g_1-1)}{\beta^2} - \frac{6}{\beta}d_1\right\},\; g_{10} = \frac{1}{g_1}\left\{-1 + (1-\beta)d_1\right\},\; g_{11} = \frac{1}{g_1}\left\{2 - 2(3-\beta)d_1\right\},$$

$$g_{12} = \frac{1}{g_1}\left\{6d_1\right\},\; g_{20} = \frac{1}{g_1}\left\{-\frac{1-\beta}{\alpha}\right\},\; g_{21} = \frac{1}{g_1}\left\{\frac{2(3-\beta)}{\alpha}\right\},\; g_{22} = \frac{1}{g_1}\left\{-\frac{6}{\alpha}\right\}.$$

Finally, substituting Eqs. (29) into (31) and the variance and covariance terms into Eq. (10) yields

$$\begin{aligned}
Var(R) &= \frac{n^2}{\alpha^2}\left(q^{-1}-1\right)^{2(1-\beta)} q^{2(n+1)} Var(x_0) \\
&+ \frac{n^2}{\alpha^2\beta^2}\left(q^{-1}-1\right)^2 \left\{(q^{-1}-1)^{-\beta} - 1\right\}^2 q^{2(n+1)} Var(\alpha) \\
&+ \frac{n^2}{\beta^4}\left(q^{-1}-1\right)^2 \left\{-(q^{-1}-1)^{-\beta} + 1 - \beta\log(q^{-1}-1)\right\}^2 q^{2(n+1)} Var(\beta) \\
&+ 2\frac{n^2}{\alpha^2\beta}\left(q^{-1}-1\right)^{2-\beta} \left\{(q^{-1}-1)^{-\beta} - 1\right\} q^{2(n+1)} Cov(x_0,\alpha) \\
&+ 2\frac{n^2}{\alpha\beta^2}\left(q^{-1}-1\right)^{2-\beta} \left\{-(q^{-1}-1)^{-\beta} + 1 - \beta\log(q^{-1}-1)\right\} q^{2(n+1)} Cov(x_0,\beta) \\
&+ 2\frac{n^2}{\alpha\beta^3}\left(q^{-1}-1\right)^2 \left\{(q^{-1}-1)^{-\beta} - 1\right\}\left\{-(q^{-1}-1)^{-\beta} + 1 - \beta\log(q^{-1}-1)\right\} q^{2(n+1)} Cov(\alpha,\beta)
\end{aligned} \quad (45)$$

Summary and Conclusions

In this study, to evaluate risk of failure of hydraulic structure, the derived variances of the risk of failure for the 3-parameter generalized logistic (GL) distribution are presented based on method of moments (MOM), maximum likelihood (ML), and probability weighted moments (PWM). The asymptotic variances of risk of failure of the MOM, ML, and PWM for the GL distribution are derived as functions of the design life, return period, and sample size.

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