Value of Regional Information using Bulletin 17B and LP3 Distribution

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To improve the accuracy of quantile estimators, Bulletin 17B recommends a number of procedures to improve at-site estimators using regional information. Several procedures, including two recommended by the Bulletin, are considered here. Because the data available at a site is generally limited, the skewness estimator can be particularly unstable. When fitting the LP3 distribution, Bulletin 17B recommends combining the station skew with a regional skew using the inverse of their mean square errors as weights. Previous studies have demonstrated the impact of a more precise regional skewness estimator on quantile estimator precision. To improve quantile estimates computed using short records, the Bulletin also suggests combining the at-site quantile estimate with a regional quantile estimate using their effective record lengths as the weights. Potential problems with this weighted estimator are discussed here. Two examples compare the precision of the Bulletin 17B weighted quantile estimator to several alternative estimators which employ different combinations of at-site and regional information, including an index flood procedure which did poorly. The simple Bulletin 17B weighting of at-site and regional regression quantile estimates performs nearly as well as more complex alternatives, and for short records provides a substantial improvement in quantile accuracy. However, when the regional standard deviation and skew are very informative and the regional mean estimator is relatively imprecise, more accurate estimates can be obtained by weighting each of the three sample moments separately with regional estimators of those same statistics.

Introduction

A large portion of the U.S. population, infrastructure, and industry is located in flood prone areas. Floods cause an average of nearly 100 deaths and cost roughly \$2.3 billion annually in the U.S. Accurate estimates of the magnitude and frequency of flood flows are needed for the design of water-use and water-control projects, for floodplain definition and management, and for the design of transportation infrastructure such as bridges and roads. Unfortunately, the accuracy of flood quantile estimates are constrained by the data available at a site: record lengths are often limited to 100 years, and are typically less than 30 years. This paper considers the use of regional skew information and regional quantile estimates to improve the accuracy of at-site quantile estimates.

The current methodology recommended for flood frequency analyses by U.S. Federal agencies is presented in *Bulletin 17B* (B17) [IACWD, 1982]. B17 recommends fitting a log-Pearson type 3 (LP3) distribution to a systematic flood record using three sample moment estimators computed after a log-transformation of the data. However, with short records the skew can be unstable. Therefore, based on research by Beard [1974] and Tasker [1978], B17 recommends combining the station skew with a regional skew to obtain a more accurate skewness estimator. This paper summarizes results of previous studies which consider regional skew estimation, the value of regional skew information in improving quantile estimates, and how to appropriately combine at-site and regional skew information.

Bulletin 17B also suggests procedures to improve quantile estimates computed using short records by employing a regional quantile estimator. Assuming the at-site and regional quantile estimates are independent, the recommended procedure is to weight the two estimates using their effective record lengths as the weights. This paper discusses potential problems with this weighting scheme, and compares the B17 weighted quantile estimator to several alternative LP3 quantile estimators that make use of different combinations of at-site and regional information.

Use of Regional Skew Information

The station skew computed from a flood record of modest length is sensitive to extreme events. To improve the accuracy of the skewness estimator, B17 recommends obtaining a weighted skew coefficient G_w by combining the sample (atsite) skew G with a regional skew \overline{G} using the linear equation:

$$G_{w} = W G + (1 - W)\overline{G} \tag{1}$$

where

$$W = MSE_{\bar{G}} / (MSE_{G} + MSE_{\bar{G}})$$
⁽²⁾

Here MSE_G is the estimated mean square error (equal to variance plus bias-squared) of the sample skew, and $MSE_{\overline{G}}$ is the mean square error of the regional skew. This weight yields the minimum MSE skewness estimator provided the at-site and regional skew estimates are uncorrelated [Griffis, 2003].

Bulletin 17B recommends approximating the MSE_G as a function of the sample skew G and the record length N using the equation provided therein. That equation is based on the Monte Carlo study reported in Wallis et al. [1974], and yields relative errors as large as 10% within the hydrologic region of interest [Griffis, 2003]. Griffis et al. [2004] provide an alternative approximation which is substantially more accurate than the approximation provided by B17, and it is consistent with the asymptotic variance for G provided by Bobee [1973].

The regional skew is generally assumed to be an unbiased estimator of the true skew γ so that $MSE_{\bar{G}}$ equals the variance of \bar{G} . Estimates of the regional skew and its variance are obtained from a separate regional analysis such as that described by B17, McCuen [1979] or Reis et al. [2005]. Regional skew values may also be obtained from the skew map provided in B17; however, the reported variance of 0.302 associated with these skew estimates may be too large. Several U.S. Geological Survey studies have employed Weighted Least Squares regression as recommended by Tasker and Stedinger [1986] to estimate a regional skew (for example, Rasmussen

and Perry [2000], and Pope et al. [2001]); others have employed Generalized Least Squares regression as described by Reis et al. [2005]. All of these studies suggest the variance of good regional skew models should be 0.10 or less (corresponding to an effective record length of 50 or more years), and not 0.30 (corresponding to an effective record length of 16 years) as suggested by the B17 skew map. Given that typical record lengths are 15-70 years, the differences in the computed effective record length are very important in the computation of a weighted skew.

Griffis and Stedinger [2004] quantify the value of regional skew information by evaluating the benefit of reducing the regional skew variance, and consider how best to combine the sample skew with the regional skew. The MSE-skew weight in eqn. (2) can yield the minimum MSE skewness estimator, but does not provide the minimum MSE quantile estimators except when the true at-site skew is zero. Griffis and Stedinger [2004] derive an optimal-quantile weight which yields minimum MSE quantile estimators. Monte Carlo results illustrate the value of using an informative regional skew and compare the two weighting schemes. For reasonable values of the regional skew, the MSE of quantile estimators is reduced when the sample skew is combined with an informative regional skew. Modest improvements in the MSE of quantile estimates are obtained using optimal quantile-weight rather than the MSE-skew weight. When the regional skew is actually very informative, there is a large loss of efficiency for positively skewed populations when either weight is incorrectly computed using a regional skew estimation error of 0.302 as recommended by the B17 skew map.

Weighting of Independent Estimates

Bulletin 17B also suggests a procedure to improve quantile estimates computed using short records by employing a regional quantile estimator obtained using either prediction equations or the index flood method. Assuming the at-site quantile estimator X and the regional quantile estimator Y are independent, the recommended procedure is to combine the estimators using their effective record lengths as the weights:

$$Z = \frac{N_X X + N_Y Y}{N_X + N_Y} \tag{3}$$

Bulletin 17B [pp. 8-1 and 8-2] demonstrates that the resulting quantile estimator Z has a smaller variance than either the at-site or regional quantile estimator provided their variances V_X and V_Y are inversely proportional to the record lengths N_X and N_Y from which they were computed. Several U.S. Geological Survey studies recommend this quantile weighting to improve flood quantile estimates (see for example, Pope et al. [2001], and Walker and Krug [2003]).

Unfortunately, the simple quantile weighting in eqn. (3) may not yield the most accurate quantile estimate. The variance of the at-site quantile estimate is dominated by the error in the sample standard deviation and skew, though the effect of the error in the skew can be reduced when regional skew information is employed as recommended by B17. On the other hand, Lettenmaier and Potter [1985] observe that the error in regional quantile estimates is dominated by uncertainty in estimates of the at-site mean. Adopted parameter values in Stedinger and Tasker [1985, 1986] describe a belief that regional models of the standard deviation are much more precise

than those for the mean. Fill [1994, p. 58] observes that "even with relatively short record lengths, one can obtain reliable at-site estimates of the mean"; he suggests a regional index flood estimator which uses at-site information for the mean and separate regional analyses for the standard deviation and the skew. Similar recommendations are made by Kuczera [1982a,b], Lettenmaier and Potter [1985], Stedinger and Lu [1985], and Hosking and Wallis [1997]. Such an approach may result in more accurate quantile estimates than the quantile weighting in eqn. (3) recommended by B17.

Evaluation of Weighting Methods. The following analysis investigates the accuracy of the quantile weighting in eqn. (3) relative to alternative estimators which employ different combinations of at-site and regional information to obtain an estimate of the logarithm of the 99th quantile ($Q_{0.99}$). The following estimators are compared here:

- 1. Bulletin 17B At-site Uses at-site sample moments with regional skew information to estimate $log(Q_{0.99})$ as recommended by B17; no weighting with regional regression quantile estimate.
- 2. Regional Regression Uses GLS regression [Tasker and Stedinger, 1989] to obtain a model to estimate $log(Q_{0.99})$ as a function of basin characteristics; no weighting with *Bulletin 17B* at-site quantile estimate.
- 3. Quantile-Weighted Uses eqn. (3) to combine *Bulletin 17B* at-site and regional regression quantile estimates. (Effective record length computations are described in Appendix A.) This represents the quantile-weighting procedure recommended by B17.
- 4. Index Flood Uses the at-site mean μ_s with separate models for the regional standard deviation σ_R and the regional skew γ_R . The at-site mean and regional moment estimates are combined to obtain:

$$\log(Q_{0.99}) = \mu_{s} + \sigma_{R} K_{0.99}(\gamma_{R})$$
(4)

where $K_{0.99}$ is a P3 frequency factor computed as a function of the regional skew γ_R . This is not the traditional L-moment index flood estimator described in Hosking and Wallis [1997].

5. Moment-Weighted (2) – Uses a weighted mean μ_W and a weighted standard deviation σ_W combined with a regional skew γ_R as follows:

$$\log(Q_{0.99}) = \mu_W + \sigma_W K_{0.99}(\gamma_R)$$
(5)

where $K_{0.99}$ is a P3 frequency factor computed as a function of the regional skew γ_R . The weighted mean μ_W and standard deviation σ_W are computed as:

$$\mu_{W} = \frac{n \,\mu_{S} + m \,\mu_{R}}{n + m}; \qquad \sigma_{W} = \frac{n \,\sigma_{S} + p \,\sigma_{R}}{n + p} \tag{6}$$

Here n is the at-site record length, and m and p are the effective record lengths of the regional mean and standard deviation models, respectively. (Effective record length computations are described in Appendix A.)

6. Moment-Weighted (3) – Uses a weighted mean μ_W and a weighted standard deviation σ_W from eqn. (6), and a weighted skew γ_W from eqn. (1) to estimate

 $log(Q_{0.99})$ using eqn. (5) wherein $K_{0.99}$ is now computed using the weighted skew γ_W . Note that these weights do not produce the minimum variance quantile estimator because they neglect possible correlation among μ , σ , and γ across sites with the same basin characteristics as well as sampling correlation among the atsite estimators of those statistics.

To evaluate the accuracy of these estimators, reasonable values of the log space moments are needed. The parameter values employed in this analysis are based on previous studies. For a partition of the U.S. into 14 regions, Landwehr et al. [1978] report mean regional skewness values in the range [-0.4, +0.3]. Using these 14 values, reasonable estimates of the population regional skew values are obtained by applying the bias correction factor $\gamma = (1+6/N)G$ [Tasker and Stedinger, 1986]. The average of these 14 unbiased skew estimates is roughly -0.1 and is employed in this study as the regional log space skew γ_R . The results of Landwehr et al. [1978] also indicate that with base-*e* conversions the average regional log space standard deviation in the U.S. is on the order of 0.5; this value is employed here for σ_R . For this analysis, the values of the regional and at-site mean are inconsequential because they do not affect the variance of the log space quantile estimators. (Variance computations are described in Appendix A.)

Reasonable values of the variances of the regional models are also needed. In this analysis, two cases are considered. The two cases differ in the precision of the regional models for the standard deviation and skew. Case 1 considers the impact of more precise regional models of σ_R and γ_R , whereas the models of σ_R and γ_R employed in Case 2 have a larger variance of prediction. For each case, Table 1 presents the values of the regional moments (μ_R , σ_R , and γ_R), and the variance and corresponding effective record length of each of the regional models. The table also includes the precision of a regional regression model for the logarithm of the 100-year event, $\log(Q_{0.99})$. For each regional model, the effective record length is computed by equating the variance of prediction of the regional model to the variance of a pure at-site estimator. Thus, a regional model with an effective record length n_e is equivalent to an at-site estimator based on n_e years of data.

Quantity	μ_R	σ_{R}	γ_{R}	$log(Q_{0.99})$
	Case 1			
Moment Estimate		0.50	-0.10	
Variance of Prediction	0.035	0.001	0.020	0.043
Effective Record Length (years)	7	125	300	36
	Case 2			
Moment Estimate		0.50	-0.10	
Variance of Prediction	0.035	0.004	0.100	0.069
Effective Record Length (years)	7	30	60	22

Table 1: Regional moment estimates, variance of prediction, and effective record length of regional regression models for four statistics

The variances of the regional moment estimators were selected based on the results of previous studies. Stedinger and Tasker [1985] indicate that the variance of the regional mean μ_R is likely to be in the range of 0.01 to 0.06. Here Var[μ_R] = 0.035 is employed. Stedinger and Tasker [1985] also suggest the variance of the regional standard deviation is on the order of Var[σ_R] = Var[μ_R]/16 \approx 0.002; values of this magnitude are employed here. For the variance of the regional skew, a value of 0.10 is adopted for Case 2 based on the results of several studies (as discussed above). In some cases, values on the order of 0.020 were also obtained; this value is adopted for Case 1. The variances employed in Case 2 are believed to represent values typically observed in practice, while Case 1 investigates the impact of highly informative regional models for the standard deviation and skew.

Figures 1 and 2 display the estimated log space variance of each quantile estimator on a log scale as a function of the at-site record length n for Cases 1 and 2, respectively. (Computations of estimator variances are described in Appendix A.) For the purposes of computing sampling variances, the results assume the true population moments for the site equal the regional moments. This should be true on average.



Figure 1: Estimated variance of alternative estimators of $ln(Q_{0.99})$ for Case 1 (more precise regional models) as a function of at-site record length

For small *n*, in both cases the value of regional information in improving the quantile estimate rather than simply using the *Bulletin 17B* at-site quantile estimator is clearly evident. The Quantile-Weighted and Moment-Weighted (3) estimators noticeably improve the quantile estimate for n < 75. When the regional skew is very informative [Case 1], the Moment-Weighted (2) estimator performs as well as the Moment-Weighted (3) estimator for n < 75. In Case 2, the Moment-Weighted (2) estimator is competitive for n < 20.



Figure 2: Estimated variance of alternative estimators of $ln(Q_{0.99})$ for Case 2 as a function of at-site record length

The Moment-Weighted (3) estimator performs as well as or better than the other estimators for all n when the regional standard deviation and skew are very informative [Case 1]. For cases more typical in practice [Case 2], the Quantile-Weighted estimator actually outperforms the Moment-Weighted (3) estimator for all n; however, the difference between the variances of the estimators is quite modest. As n increases, the variance of the Quantile-Weighted, *Bulletin 17B* at-site, and Moment-Weighted (3) estimators all go to zero, whereas the variance of the Moment-Weighted (2) estimator will approach an asymptote because the regional skew is not updated with the at-site skew.

For small *n*, the Index Flood estimator was expected to be more precise than the Quantile-Weighted estimator. The variance of the Index Flood estimator is dominated by the error in the at-site mean for small *n*. When the regional standard deviation and skew are very informative [Case 1], the Index Flood estimator is competitive for 15 < n < 60. However, in both cases, the variance of the Index Flood estimator approaches an asymptote due to the error in both the regional standard deviation and the regional skew. In Case 2, error in the regional standard deviation contributes nearly half of the variance of the regional regression estimator, which is why the Quantile-Weighted estimator does so well relative to the Moment-Weighted (3) estimator: the standard deviation error is critical for both estimators. Moreover, the Quantile-Weighted estimator employs the *Bulletin 17B* at-site quantile estimator dominates the at-site skew estimator in Case 1, and for small *n* in Case 2; thus, the Quantile-Weighted

estimator combines two quantile estimators whose error is dominated by the error in the standard deviation.

The results in Figures 1 and 2 were obtained under the assumption that all errors of the regional moment models (μ , σ , and γ) are independent of one another. Previous regional regression studies [Thomas and Benson 1970; Tasker and Stedinger 1989] suggest this may not be true. Supplemental analyses indicated that for different parameter sets, adding such correlation did not affect the relative performance of the estimators. Thus, the Moment-Weighted (3) estimator is preferred, although the Quantile-Weighted estimator recommended by B17 generally performs about as well.

Conclusions

A computational study demonstrates the relative advantage of regional flood quantile estimators at sites with short records. The simple weighting of at-site and regional regression quantile estimates recommended by *Bulletin 17B* performs nearly as well as more complex alternatives, and for short records provides a substantial improvement in quantile accuracy. However, more accurate estimates would be obtained using a Three-Moment-Weighted estimator when the regional standard deviation and skew are very informative.

Appendix A. Variance of Quantile Estimators

As a function of the at-site record length n, Figures 1 and 2 display the log space variance of six LP3 quantile estimators which employ different combinations of atsite and regional information. The variances of the six estimators were estimated analytically as follows:

- 1. *Bulletin 17B* At-site Chowdury and Stedinger [1991, eqn. 19] provide a first-order approximation of the variance with a weighted skew.
- Regional Regression The variance of the regional regression quantile estimator is equivalent to the variance of prediction for the regional regression model at the site in question obtained using GLS regression [Tasker and Stedinger 1989, eqn. 17] whose estimator of the sampling covariance matrix neglects the error in the computed skew. For this example, it is assumed that the errors of the regional moment models are independent, so to first-order the variance of the regional quantile model is

$$\operatorname{Var}[\hat{y}_{0.99}] = \operatorname{Var}[\mu_R] + K_{0.99}^2 \operatorname{Var}[\sigma_R] + \sigma^2 \left(\frac{\partial K_{0.99}}{\partial \gamma}\right)^2 \operatorname{Var}[\gamma_R]$$
(7)

3. Quantile-Weighted – Assuming the at-site quantile estimate y_s and regional quantile estimate y_R are independent, the variance is given in B17 [eqn. 8-3]:

$$\operatorname{Var}[\hat{y}_{0.99}] = \frac{\operatorname{Var}[y_S] \operatorname{Var}[y_R]}{\operatorname{Var}[y_S] + \operatorname{Var}[y_R]}$$
(8)

Here the Bulletin 17B estimator is represented by the "at-site" quantile estimate.

4. Index Flood – Using $\hat{y}_{0.99} = \hat{\mu}_s + \sigma_R K_{0.99}(\gamma_R)$, to first order the variance is

$$\operatorname{Var}[\hat{\mathbf{y}}_{0.99}] = \operatorname{Var}[\hat{\boldsymbol{\mu}}_{S}] + K_{0.99}^{2} \operatorname{Var}[\boldsymbol{\sigma}_{R}] + \boldsymbol{\sigma}^{2} \left(\frac{\partial K_{0.99}}{\partial \gamma}\right)^{2} \operatorname{Var}[\boldsymbol{\gamma}_{R}]$$
(9)

wherein it is assumed the errors in σ_R and γ_R are independent of each other, and independent of $\hat{\mu}_s$.

5. Moment-Weighted (2) – The variance is similar to eqn. (9), however, the weighted mean $\hat{\mu}_w$ and the weighted standard deviation $\hat{\sigma}_w$ are used in place of the at-site mean $\hat{\mu}_s$ and the regional standard deviation σ_R , respectively. The variance of the Moment-Weighted (2) estimator also includes the term $\{2K_{0.99}n^2/[(n+m)(n+p)]\}$ Cov $[\hat{\mu}_s, \hat{\sigma}_s]$, where *m* and *p* are the effective record lengths of the regional mean and standard deviation models, respectively. The variance of the weighted mean and standard deviation are computed as

$$\operatorname{Var}[\hat{\mu}_{W}] = \frac{\operatorname{Var}[\hat{\mu}_{S}] \operatorname{Var}[\mu_{R}]}{\operatorname{Var}[\hat{\mu}_{S}] + \operatorname{Var}[\mu_{R}]}; \qquad \operatorname{Var}[\hat{\sigma}_{W}] = \frac{\operatorname{Var}[\hat{\sigma}_{S}] \operatorname{Var}[\sigma_{R}]}{\operatorname{Var}[\hat{\sigma}_{S}] + \operatorname{Var}[\sigma_{R}]}$$
(10)

It is assumed that the errors in the regional moment estimators are all independent of one another, and that they are independent of the at-site moment estimates.

6. Moment-Weighted (3) – The variance is similar to that of the Moment-Weighted (2) estimator, however, the weighted skew ŷ_w is used in place of the regional skew γ_R, and Var[ŷ_w] = W²Var[ŷ_s]+(1-W)²Var[γ_R] where W is given by eqn. (2). The variance of the Moment-Weighted (3) estimator also includes the term 2WK_{0.99}(∂K_{0.99}/∂γ)[nσ/(n+p)] Cov[ô_s, ŷ_s]. Again, it is assumed the errors in the regional moment estimators are all independent of one another, and that they are independent of the at-site moment estimates.

Bobee [1973] provides approximations for the variance and covariance of the at-site moments μ_s , σ_s , and γ_s . For each regional model, the effective record length is computed by equating the variance of prediction of the regional model to the variance of a pure at-site estimator. Thus, a regional model with an effective record length n_e is equivalent to an at-site estimator based on n_e years of data.

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