Long memory of rivers from spatial aggregation

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[1] Long memory is a hydrological property that can lead to prolonged droughts or the temporal clustering of extreme floods in a river. Analyses of 28 long (up to 145 years), continuous instrumental runoff series from six European, American, and African rivers reveal that this effect increases downstream. Simulations reproduce the increase qualitatively and show that a river network aggregates short-memory precipitation and converts it into long-memory runoff. In view of projected changes in climate and the hydrological cycle, these findings show that decadal-scale variations in drought or flood risk can be predicted for individual rivers, with higher predictability downstream. Spatial aggregation may also explain the emergence of long memory in other networks, such as the brain or those formed by computers.


1. Introduction

[2] Temporal variations in the occurrence of extreme river floods [Mudelsee et al., 2003] or droughts can be caused [Bunde et al., 2005] by a property of runoff time series that is denoted as long-range dependence or long memory; such records are said to exhibit, after its discoverer [Hurst, 1951], the “Hurst phenomenon.” This refers to the ability of a hydrological system to “remember” past states over long time (decades). However, few physical mechanisms (climate instationarities [Potter, 1976] and storage cascade mechanisms [Klemes, 1974, 1978]) have been presented to explain how long memory emerges, and none has been quantitatively tested because of the scarcity of long records. This hampers modeling and prediction attempts. In this paper, I propose an explanation based on spatial aggregation of precipitation contributions in a river network. To test this, I use long instrumental runoff records and study the long-memory parameter as a function of the basin size.

2. Hypothesis

[3] Many meteorological variables have noise components that can be described as simple first-order autoregressive (AR(1)) processes [Gilman et al., 1963], where a value depends only on its own immediate past plus a random component. The autocorrelation function (acf) or “memory” of such variables decays exponentially (fast), as \( \sim a^h \), where \( a \) is the autocorrelation coefficient (between 0 and 1) and \( h \) is the time lag [Beran, 1994]. Studying runoff time series from the river Nile and other hydrological records, Hurst [1951] found deviations from the short-memory exponential behavior. This inspired the development of statistical long-memory processes [e.g., Mandelbrot and Wallis, 1969], for which the autocorrelation function decays hyperbolically (slowly), as \( \sim h^{2d-1} \), where \( d \) (less than 0.5) is the long-memory parameter. (Hurst defined a coefficient equal to \( d + 0.5 \).)

[4] In theory, both acf \( \sim h^{2d-1} \) and acf \( \sim a^h \) attain zero values only for \( h = \infty \); in practice (finite data sizes and nonzero measurement errors), the empirical acfs cannot be distinguished from zero already for finite \( h \), and this \( h \) value may be considerably larger for long-memory than for short-memory processes. A flexible long-memory model is the ARFIMA type [Hosking, 1984], which offers exact maximum likelihood estimation and model suitability tests [Beran, 1994]. Error bars for the estimated ARFIMA parameter \( h \) indicate also how strong \( d \) differs from zero, that means, they enable to distinguish between long and short memory. The alternative estimation techniques, namely rescaled range analysis and detrended fluctuation analysis, are inferior in this regard, as has been shown by Hosking [1984] and Maraun et al. [2004], respectively.

[5] An explanation of the Hurst phenomenon in river runoff requires not only a mathematical model, but also a description based on the physical-hydrological properties of the system [Koutsoyiannis, 2005a, 2005b]. As physical cause of long hydrological memory, changes in the mean of a meteorological variable have been suggested, for example, jumps as a result from abrupt climate changes [Potter, 1976]. The problem is that the jumps are not allowed to occur at an arbitrary rate but the no-jump probability has to be roughly inversely proportional to the length of an interval with no jumps, for which no reason seems readily apparent [Klemes, 1978]. River basins (Figure 1), forming a network of tributaries, confluences, and reservoirs that has been geometrically characterized as a fractal object [Rodriguez-Iiturbe and Rinaldo, 1997] offer another line of explanation. Consider a unit \( j \) of area \( A_j \), that is, a single, homogenous reservoir with a linear release rule, \( Q_j = k_j s / \Delta t \), where \( Q_j \) is outflow, \( k_j \) is a dimensionless positive constant describing the storage strength, \( s_j \) is basin storage volume, and \( \Delta t \) is the discretization time step. If the input to the reservoir \( j \), given by precipitation minus evaporation, is a random series, which is assumed to be fulfilled for \( \Delta t = 1 \).
To test the aggregation hypothesis, I studied the long-memory parameter in dependence of the basin size downstream individual rivers. The idea of such $d(A)$ estimations is that with increasing $A$ also the number $m$ of short-memory runoff contributions $Q_j$ grows. Thereby should also $d$ increase, from zero ($m = 1$) to a saturation level below 0.5 ($m$ large). The mathematical requirements for this $d(A)$ behavior have previously been verified. The saturation level is mainly a function of the upper bound, $a_{\text{max}}$, of the distribution of the $a_j$ [Granger, 1980; Linden, 1999], and model simulations (Figures 2g and 2h) demonstrate that $d(A)$ saturation sets in for $m$ around hundred (corresponding to $A = 40,000 \text{ km}^2$ for an average $A_j$ of $20 \times 20 \text{ km}^2$). The reason why to study $d(A)$ for each river separately, thereby expecting that $d(A)$ at point 4 (Figure 1) is higher than $d(A)$ at point 3 and so forth, is as follows. Rivers scatter in their lengths, basin sizes, storage properties $k_p$, precipitation occurrence within their basin area and hence their $d(A)$ saturation levels (Figure 2). This scatter disappears when analyzing rivers separately. That means, interriver variability does not blur the $d(A)$ curves in my analyses. To further suppress variability from temporal changes, the runoff records from the gauge stations along a river (points 1 to 4 in Figure 1) are required to span a common time interval.

For the data analyzed (Figure 2) the water that passes through point 1, passes also through point 2, and so forth, provided that water loss from evaporation or infiltration is negligible. This test design should make the $d(A)$ curves a sensitive indicator of whether spatial aggregation causes the long memory. A $d(A)$ increase with saturation would be a positive result. The absence of this behavior could reflect violated assumptions in the formulation of the model of cascading linear reservoirs (e.g., too strong nonlinearities) or inadequate data. The runoff records must be long enough to yield acceptably small $d$ estimation uncertainties [Hurst, 1951; Montanari et al., 1997; Koutsoyiannis, 2005a]; the series should be without temporal gaps, which could bias the $d$ estimation [Hwang, 2000]; and the gauge stations along a river should not be too few and cover a large $A$ range to capture the $d(A)$ increase. These demands severely limit the number of instrumental records that qualify for the $d(A)$ estimation. Adopting a minimum length of 70 years (840 monthly runoff values), only a handful of rivers (Weser, Elbe, Rhine, Colorado, Mississippi, and Nile) could be taken from the large database of the Global Runoff Data Centre (Koblenz, Germany). Particularly the Weser [Zimmermann et al., 2000], with four gauge stations and records of 145 year length each, is considered as of high data quality. The auxiliary material lists rivers, stations, time intervals, and basin sizes.

Logarithmic transformations [Klemes, 1978; Montanari et al., 1997] brought the right-skewed runoff records to approximately normal shape. Subtraction of daywise (monthwise) long-term averages from daily (monthly) records removed annual cycles. Linear detrending removed instationarities owing to climatic changes. Downsampling to 30-day segments of daily records allowed comparability with monthly records. Long-memory ARFIMA($1, d, 0$) models with an autoregressive component [Hosking, 1984] were fitted by exact Gaussian maximum likelihood [Doornik and Ooms, 2003] to the data. Comparisons with ARFIMA($0, d, 0$) fit results attested in all cases that including the autoregressive component improved the fit quality. Bootstrap simulations [Doornik and Ooms, 2003] yielded error bars and significances of $d$ estimates. See auxiliary material for numerical results.

**4. Results**

For most records, the long-term trends were upward, which is consistent with previous findings for other regions [Peterson et al., 2002], and perhaps an indirect (via global
warming) or direct (via suppressed plant transpiration) effect of changes in atmospheric carbon dioxide concentration [Gedney et al., 2006]. However, linear detrending had only minimal (within error bars) influence on the $d(A)$ curves, which was found also when employing stepwise trend functions [Mudelsee, 2000]. This argues against explanations [Potter, 1976] of the Hurst phenomenon via climate instationarities.

**Figure 2.** Long-memory parameter, $d$, in dependence on basin size, $A$. Vertical bars are $1 - \sigma$ standard errors. (a) Weser, January 1857 to April 2002; (b) Elbe, November 1899 to October 1990; (c) Rhine, January 1920 to December 2003; (d) Colorado, April 1934 to September 2003; (e) Mississippi, April 1906 to September 2003; (f) Nile, January 1912 to December 1982; (g, h) model. The runoff model generated records by the aggregation of $m$ first-order, unit variance autoregressive Gaussian processes, $Q_j$. Each $Q_j$ series had a data size of 1000, corresponding to a record length of 83 years at monthly resolution. Here $m$ varied logarithmically from 2 to 100, corresponding to $A$ between 800 and 40,000 km$^2$ under the assumption of a mean unit area of $A_j$ of 400 km$^2$. For each $m$, 400 simulations of the procedure aggregation-estimation were performed. The autocorrelation coefficient of the $Q_j$ process was thereby drawn from a uniform distribution [Linden, 1999] over (0.0; $a_{\text{max}}$). Shown (solid symbols) are average ± standard error of each $d$ estimate over the simulations. Figure 2h also shows (open symbols) the result from simulations where the variance of $Q_j$ is not constant over $m$ but itself a zero-mean, unit variance Gaussian random variable.
The results for the river Weser (Figure 2a) show a significant increase of \( d \) with \( A \) in the upper river part (small \( A \)), while for the station in the lower part (large \( A \)) there is no change within error bars. This increase-saturation behavior is as expected under the aggregation hypothesis. A permutation test finds that by chance such an increase occurs with a probability of 1/24, equivalent to a confidence level of 95.8%.

The \( d(A) \) curves for the two other central European rivers Elbe and Rhine agree with the aggregation explanation (Figures 2b and 2c). In case of the Elbe, the station Turice on the tributary Jizera was included because no long record was available from the upper Elbe. Although the Turice record is slightly shorter than the other Elbe records (80 years versus 91 years), its \( d \) is significantly smaller than the \( d \) values for stations further downstream of the Elbe (Figure 2b). Those points \((A > 50,000 \text{ km}^2)\) appear to sit close to the saturation part of the curve, with only small \( d \) increases and one insignificant \( d \) inversion (station Neu-Darchau). Still, the Elbe \( d(A) \) increase has a permutation test confidence of 96.7%. The four Rhine gauge stations cover a large \( A \) range (Figure 2c) and exhibit a clear increase of the \( d(A) \) graph in the upper to middle river parts (91.7% confidence). The Colorado river (USA) has only three stations covering a long enough period (70 years). Its \( d(A) \) curve (Figure 2d) displays an increase, which, however, is significant only at a level of 83.3% due to the small number of stations. The apparent absence of saturation for \( A \) up to 445,000 \text{ km}^2 (gauge station below Hoover Dam) might indicate larger areas \( A \) in the Colorado basin, which would be compatible with enhanced spatial dependence of meteorological–hydrological properties, in comparison with central Europe (Weser, Elbe, and Rhine). However, it appears too speculative to relate this to the large size of the Hoover Dam reservoir without a detailed hydrological model analysis.

The \( d(A) \) curve combined from the Blue Nile and the Nile (Figure 2f) shows an increase, which scatters somewhat due perhaps to the relatively short records (70 years). For a small basin size, a nonsignificant \( d \) was found (Blue Nile, Roseires Dam). An exception from the monotonic \( d(A) \) increase is nilometer Hudeiba, but this deviation is within error bars. Although basin sizes are considerably larger than those of the other rivers, no clear \( d(A) \) saturation was found. Data from nilometer Roda at Cairo, analyzed by Hurst [1951], could resolve this, but this record has only annual resolution, and a comparison of \( d \) values would likely be biased. Interestingly, the low \( d \) values (0.2 and less) for the Blue Nile or Nile (Figure 2f) contrast with the \( d \) values from the lower White Nile, which are close to 0.5 (see auxiliary material). In case of the White Nile, the aggregation hypothesis might not be applicable because in that region (Sudd swamp) strong evaporation occurs. After the confluence of the Blue Nile and the White Nile at Khartoum, it seems that the memory of the Nile is determined by the Blue Nile, while the memory of the White Nile is “lost.” This agrees with observations [El-Sebaie et al., 1997] that the Blue Nile dominates the water supply although it has a smaller basin size than the White Nile.

The \( d(A) \) curve from the Mississippi has no increase as expected under the aggregation hypothesis, but rather a decrease (Figure 2e). On the other hand, despite the considerable length of the records (97 years), the errors are wide enough to explain this decrease as a coincidence of values in the \( d(A) \) saturation region. (The slope of a weighted regression line is significantly different from zero only at \( 1 - p = 86\% \), in agreement with 83.3% from the permutation test.) One could alternatively seek to augment the aggregation model, for example by invoking jumps in the distribution of the \( q \), but this seems premature as long as no long records from the upper Mississippi confirm the \( d(A) \) decrease.

**5. Discussion and Conclusions**

Following assumptions went into the evaluation of the aggregation hypothesis. First, the runoff records are obtained from water stage measurements via stage-runoff calibrations. Although these formulas have been regularly updated and the river morphologies studied over time [Zimmermann et al., 2000; Modelsee et al., 2003], systematic uncertainties, which would propagate into \( d(A) \) errors, cannot be ruled out. However, the high test confidence levels indicate that systematic errors are small. Second, river basins need not obey a fractal geometry [Rodriguez-Iiturbe and Rinaldo, 1997] to produce long memory. As the modeling experiments (Figures 2g and 2h) clearly demonstrate, the confluence of around hundred independent short-memory contributions (unit areas \( A_j \)) is already sufficient.

Third, those contributions are interpreted as AR(1) processes generated by a linear reservoir [Klemes, 1978] that stores random precipitation input. On the one hand, the linearity assumption is supported because the time spacing considered (1 month) reduces nonlinearities like short-term unsteady flow [Klemes, 1978]. On the other, the assumption that water storage variations operated by a single basin and water losses from evaporation or infiltration are negligible, could well be violated for larger basins. Indeed, this might explain the absent (Mississippi, Figure 2e) or nonmonotonic (Nile, Figure 2f) increases of the \( d(A) \) curves.

Further, at monthly resolution precipitation should be random, without short or long memory. To test this, a gridded data set [Hulme et al., 1998] of mean monthly precipitation in 1900–1998 was analyzed in the same manner as the runoff records. The centers of the grid cells (2.5° latitude by 3.75° longitude) representing the river basins were: Weser, 52.5°N, 11.25°E; Elbe, 50°N, 15°E; Rhine, 47.5°N, 7.5°E; Colorado, 37.5°N, 112.5°W; Mississippi, 42.5°N, 97.75°W; and Nile (annual precipitation), 15°N, 33.75°E. From the six precipitation records, none had a long-memory parameter above 0.1 or different from zero at a confidence level of 95% (Weser, 94.3%; Elbe, 45%; Rhine, 88.3%; Colorado, 78%; Mississippi, 88.7%; and Nile, 45%), and none had an autoregressive component different from zero at a level above 72%. Also analyses of long rainfall records from the United States [Potter, 1979] or Italy [Montanari et al., 1996] found no or only modest long memory. Even if rainfall had a small long-memory parameter [Hurst, 1951; Bunde et al., 2005], this should not be sufficient to falsify the aggregation explanation, which would likely be required to produce the larger \( d \) values of runoff and the \( d(A) \) increases (Figure 2).
tation contributions are independent. When the spatial variability of rainfall itself is strongly time-dependent (e.g., varying types of rainfall), the theoretical $d(A)$ saturation curves (Figures 2g and 2h) may become distorted.

[17] Deeper theoretical knowledge about the long-memory properties of aggregated short-memory processes would permit to test harder the mathematical part of the aggregation hypothesis as an explanation of the Hurst phenomenon. If we assume the $m$ aggregated AR(1) processes to have identical means (zero), identical variances (unity), and either beta-distributed [Granger, 1980] or uniformly distributed [Linden, 1999] autocorrelation coefficients $a_j$, then the aggregated process can be shown to possess long memory. However, it appears likely that also under relaxed assumptions regarding the processes, long memory can emerge in the aggregation. Less the variance (Figure 2b), skewness [Anis and Lloyd, 1975], or shape [Mudelsee, 2006] of the distributions of the processes than rather their serial dependence structures [Granger, 1980; Linden, 1999] should be relevant hereby. For example, the interval bound of the $a_j$ in case of aggregated AR(1) processes has considerable influence on the $d(A)$ saturation level (Figures 2g and 2h). Because most measured precipitation and runoff records are relatively short for accurate long-memory estimations, long runoff model simulations should also be utilized.

[18] Almost all analyzed rivers confirm the aggregation hypothesis. They show $d(A)$ increases with confidence (Figure 2). An exception is the Mississippi, for which an interpretation compatible with the aggregation hypothesis is that the areas $A_j$ are small, so that only the $d(A)$ saturation range was sampled. The aggregation explanation of long memory expands the idea of Klemes [1978], who invoked cascading reservoirs. The difference is that under aggregation a $d(A)$ increase is predicted. This could be tested and verified using exceptionally long runoff records from several stations downstream individual rivers. The observed $d(A)$ increases (Figure 2) contradict previous [Hurst, 1951] claims of a constant $d$ parameter. This is supported by a recent paper [Koscielny-Bunde et al., 2006], which finds considerable $d$ variation among 41 river runoff records. Interestingly, this paper does not detect a systematic $d(A)$ dependence, likely because interriver variability was not taken into account. The aggregation explanation further refutes a previous statement [Potter, 1979] that “if long-term persistence in streamflow series has a physical basis, it must lie in the precipitation process.”

[19] Spatial aggregation, an inherent property of river networks (Figure 1), can produce long memory and constitutes a simple physical explanation of the Hurst phenomenon [Hurst, 1951]. Long memory in river runoff has considerable effects on flood risk assessment. It can lead to clustering of extreme floods [Bunde et al., 2005] and long-lasting droughts and produce temporal changes in flood risk [Mudelsee et al., 2003]. Long memory further means long-range dependence and a reduction of the number of independent observations [Koutsoyiannis, 2005a, 2005b]. As a result, error bars become wide and estimates of quantities like flood return periods become rather uncertain. Likewise, statistical tests suffer from a reduced power compared to a situation without long memory [Cohn and Lins, 2005; Rybiski et al., 2006]. Such effects have also to be anticipated in data where the aggregation is introduced artificially, namely by averaging, as for example in global temperature records [Bloomfield, 1992]. On the other hand, long-term changes in the risk of extremes such as floods can in principle be predicted for individual rivers. With the long-memory parameter, also predictability should increase downstream. The envisioned forecasting tools feed projected precipitation changes [Giorgi et al., 2001] into hydrological models, for which realistic river network properties (parameters $A_j$, $k_j$, and $s_j$) have to be available.


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References


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