APPENDIX 4 – RECONSTRUCTION METHOD

This appendix lists steps by which the set of residual tree-ring chronologies described in Appendix 3 were converted into a reconstruction of flow. The objective was a reconstructed time series of water-year flow for the sum of the Salt, Verde and Tonto rivers. After a certain step, the reconstruction procedure becomes repetitive, and is repeated for three different sub-periods of the tree-ring record. Much more detailed information, restricted for brevity to one of the sub-periods (1451-1982 model) is given in the pdf file **`recon_method.pdf`**, a manuscript in preparation for submission to a journal.

1. Residual tree-ring chronologies for the 25 sites were organized in a time series matrix. The starting point for the procedure is this time series matrix and the single time series of water-year flows for the sum of the Salt, Verde and Tonto rivers (SVT). This series is referred to as "flow" in the remainder of this appendix. The flow series covers water years 1914-2007. The tree-ring matrix covers years 1100-2005, though time coverage varies by tree-ring chronology.

2. Each chronology was filtered and scaled into a single-site reconstruction (SSR) of sum of water-year total flow by the following steps:

    a. Flow was regressed on the tree-ring chronology and its lagged values (±2 yr) by robust regression using the Huber weighting function to de-emphasize outliers (Montgomery 1990). The calibration period for the regression included the full overlap of the chronology with flow. The predictors were either the chronologies themselves or the squared chronologies (quadratic model) where appropriate to adapt to possible curvilinear relationships between tree-ring index and flow. The single-site regressions themselves were done in two steps, first estimating a non-lagged model and then entering lags as deemed appropriate in a stepwise procedure. Lags were entered only if they resulted in improved skill of prediction when the model was validated with split-sample validation and cross-validation, and only up to a possible lag of ±2 years from the year (water year) of flow. How many and which lags were allowed to enter was guided by cross-validation (leave-9-out) (Meko 1997) to ensure that a lag entered only if it resulted in improved prediction accuracy on data not used to fit the model.

    b. The long-term record of tree-ring index was substituted into the regression equation to generate the SSR of flow.

3. The 25 single-site reconstructions (SSRs) were reduced to 21 by dropping chronologies whose flow signal was insignificant (overall F of regression at α=0.05) or unstable when the model was tested with split-sample calibration/validation (Snee 1977).

4. The resulting 21 filtered and scaled chronologies, or SSRs, were organized in a time series matrix.

5. A series of exploratory regressions was run to identify subsets of chronologies that would yield strong flow reconstructions with 1) maximum length, 2) maximum accuracy, and 3) up-to-date time coverage. This analysis identified three sub-periods --1330-1989, 1451-1982, and 1736-2005 – for development of reconstruction models.

6. Reconstructed flows for each sub-period were then generated by the following steps:

    a. Those SSRs with no missing data in the sub-period were assembled in a time-series matrix

    b. A principal components analysis (Mardia et al. 1979) was run on the covariance matrix of the SSRs

    c. The scores of the first principal component (PC1) were extracted as a new tree-ring predictor. This score series is alternatively a linear combination of the SSRs and a weighted time series of filtered and scaled chronologies.

    d. Observed flows were plotted against the scores of PC1 in a scatterplot. This scatterplot is of course restricted to the overlapping period of PC1 and flows (e.g., 1451-1982) for the middle of the three sub-periods.

    e. Locally weighted regression (loess) is used to smooth the scatterplot (Cleveland 1979). The locally weighted regression was fit to the set of target points along the PC1 axis at the minimum, maximum and percentiles 5, 10, 15, …, 95 of the scores. At each of these points a neighborhood was defined based on a specified loess smoothing parameter, α. By trial-and-error, we found $\alpha = 0.6$ as a suitable setting. The tri-cube weighting function (Martinez and Martinez 2005) was used to down-weight points according to increasing distance from target point along the axis of PC1 scores. The loess estimates of flow at the target points of PC1 scores were joined by straight lines. This is the loess curve, or the smoothed scatterplot.

    f. To accommodate long-term reconstruction from the tree-ring data, the loess curve was linearly extended to the extremes (low and high) of PC1 scores in the tree-ring record. This was done simply by extending the straight line between the 5th percentile and the minimum to the left, and the straight line between the 95th percentile and the maximum to the right on the scatterplot

    g. Flows corresponding to PC1 scores from the long-term tree-ring record were linearly interpolated from the loess-smoothed scatterplot.

7. Because for a given sub-period model the variance of reconstruction errors (observed flows minus predicted flows using the loess plot) clearly increased toward higher predicted flows, an ad-hoc method was adopted to assign confidence bands to the reconstruction. This method is described in detail elsewhere (see **`recon_method.pdf`.** The main idea is that the appropriate error distribution for setting error bars for any reconstructed flow should rely mainly on errors for similar levels of reconstructed flow in the calibration period. The first step was to use cross-validation to generate calibration-period errors not biased low by tuning of the loess plot. We used a leave-9-out strategy for this cross-validation: 1) 9 sequential observations were left out, 2) the loess plot was re-estimated using the remaining data, 3) the loess plot was used to estimated the flow for the central of the left-out observations, and 4) the process 1-3 was repeated until a complete time series of "deleted residuals" was generated. (Note that for the first 4 and last 4 observations it was necessary to leave out fewer than 9 observations.) The cross-validation residuals were then re-samples using

weighted bootstrapping to estimated a confidence interval for any reconstructed flow of the long-term record. If this flow is denoted as $\hat{y}$ and the deleted residuals are $e_{(i)}$, the process is as follows:

a. Find the subset of residuals $e_{(i)}$ for the αn reconstructed flows of the calibration period nearest $\hat{y}$ in size, where *n* is the number of observations in the calibration period (the data for the scatterplot), and α=0.6. Essentially this means the cross-validation errors for the 60% of the calibration-period reconstructed flows nearest $\hat{y}$.
b. Re-sample that subset of residuals using weighted re-sampling to generate a sample of 1000 residuals. The bi-square function (Martinez and Martineez 2005) was used as weighting function.
c. Add each of these simulated residuals to $\hat{y}$ to get 1000 noise-added, or plausible "true" flows for the reconstruction year.
d. Use the empirical cumulative distribution function (cdf) of those noise-added flows to assign the confidence interval for $\hat{y}$. For example, the 0.10 and 0.90 probability points of the cdf define an 80 percent confidence interval.

Literature Cited

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