# APPENDIX 6 -- CROSS-SPECTRAL ANALYSIS

A cross-spectral analysis of flows on the Colorado River at Lees Ferry and the Salt+Verde+Tonto River (SVT) was run to summarize variation and covariation of the two flow series in the frequency domain and to identify any periodic features of the flow records. The smoothed periodogram method (Bloomfield 2000) was used to estimate spectra and cross-spectra. The method is identical to that used by Meko and Woodhouse (2005) in study of joint drought in the Upper Colorado and Sacramento River watersheds. Four separate analyses were run: 1) reconstructed flows for 1521-1964, 2) observed flows for 1914-2001, 3) reconstructed flows for 1914-1964, and 4) observed flows for 1914-1964. Results are summarized in Figures 1-4.

The long-term (1521-1964) reconstructions show strong and significant coherency across all frequency bands (Figure 1). The decadal and longer fluctuations are most coherent. The individual spectra are quite nondescript in that no strong periodicity is evident in either flow series. Some tree-ring studies (e.g., Mitchell et al. 1979; Cook et al. 1997) have suggested a bidecadal rhythm in area of drought coverage in the western United States, but the spectra plotted in Figure 1s offer no support for basin-scale expression of such a rhythm in either the Colorado or the Salt-Verde-Tonto basins. Additional spectral runs with narrower bandwidths (not shown) gave essentially the same results in terms of lack of significant periodicity. Both spectra in Figure 1s are mildly low frequency, which is consistent with weak positive first-order autocorrelation. First-order autocorrelation is greater for the Lees Ferry reconstruction ($r_1$=0.20) than for the Salt+Verde+Tonto ($r_1$=0.12), but both first order autocorrelation coefficients are significantly greater than zero at $\alpha$=0.05 in a one-sided test for the given sample size of 444 years.

The observed flows for their full overlap (1914-2001) show intervals of significant coherency at various frequency ranges, with highest coherency again at the decadal and longer time scales (Figure 2). The estimated coherency is not as uniformly significant in as for the reconstructed flows, but this characteristic is mainly an effect of sample size: the much longer period for the reconstruction attaches statistical significance to the same level of coherency that is not statistically significant for the relatively short period covered by observed flows. The individual spectra of the observed flows, like those of the reconstructions, do not show any significant periodicity, and are mildly low-frequency in appearance

The final two spectral figures (Figures 3 and 4) are included to summarize cross-spectral analysis of reconstructed flows and of observed flows for the <u>same</u> time interval (1914-1964). The spectra and coherency squared for the reconstructions strongly resemble those of the observed flows, attesting to the ability of the reconstructions to track the observed flows. For example, the spectra are mildly low-frequency, and the coherency peaks near 3 years and at the longest wavelengths. The high coherency at longest wavelengths reflects a general decline in flows from the early 1900s into the drought of the 1950s and 60s in both flow series.

The reconstruction for the SVT suggests that the observed record somewhat underestimates the persistence in flows for that record. For 1914-1964, both the observed and reconstructed flows for the SVT have near-zero first-order autocorrelation ($r_1$=0.02 for observed, $r_1$=0.04 for reconstructed). Both estimates are below the critical threshold of $r_1$=0.21 required to reject the null hypothesis of zero first-order autocorrelation (.05 $\alpha$-level, one-tailed test, $N$=51). In contrast, the long-term (1521-1964) reconstructed flows for the Salt+Verde+Tonto have a higher first-order autocorrelation ($r_1$=0.12), which for the much longer sample size of 444 years is above the critical threshold of r1= 0.08.

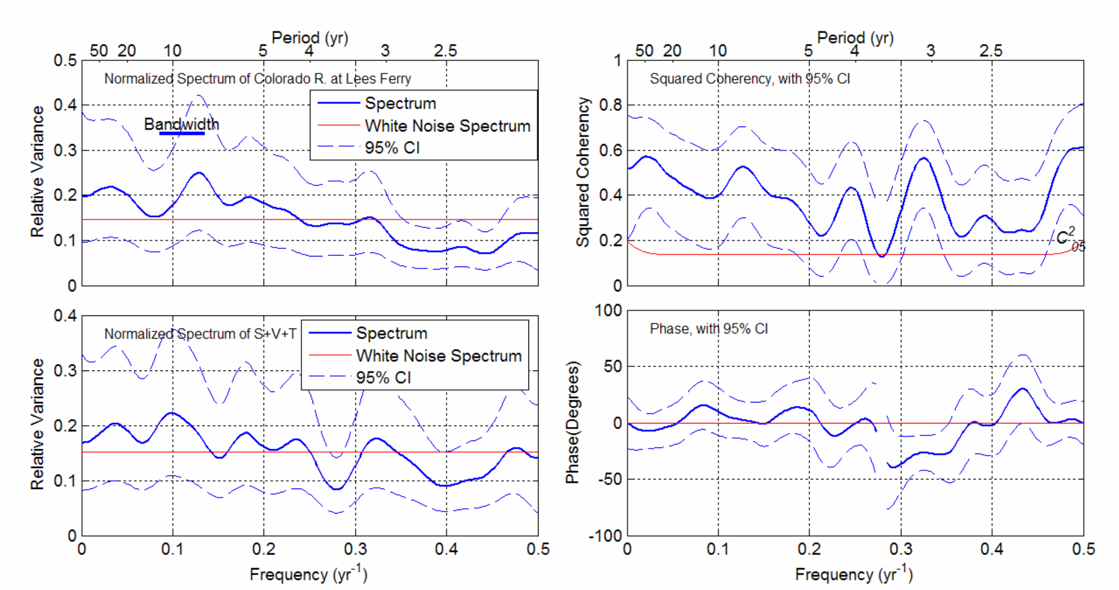


Figure 1 -- Cross-spectral analysis of reconstructed flows of Colorado River at Lees Ferry and Salt+Verde+Tonto River for full overlap period, 1521-1964. At left are the estimated spectra of individual series, with 95% confidence interval (dashed) and theoretical white noise spectrum for reference. At top right is the squared coherency with confidence interval. Horizontal line marks level of coherency for rejection of null hypothesis of zero coherency at 95% confidence level . At bottom right is the estimated phase, with confidence interval and horizontal line at zero phase. Following Bloomfield (2000), confidence interval on phase plotted only for frequency ranges where coherency is significantly greater than zero at 95% level.

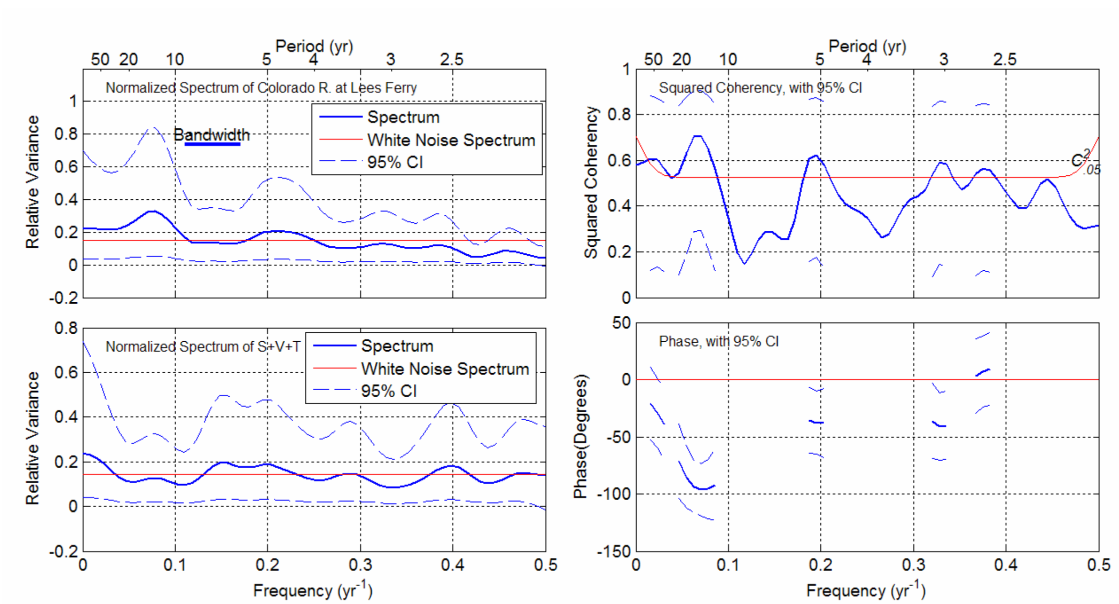


Figure 2. Cross-spectral analysis of observed flows of Colorado River at Lees Ferry and Salt+Verde+Tonto River for full overlap period, 1914-2001. Remainder of caption as in Figure 1.

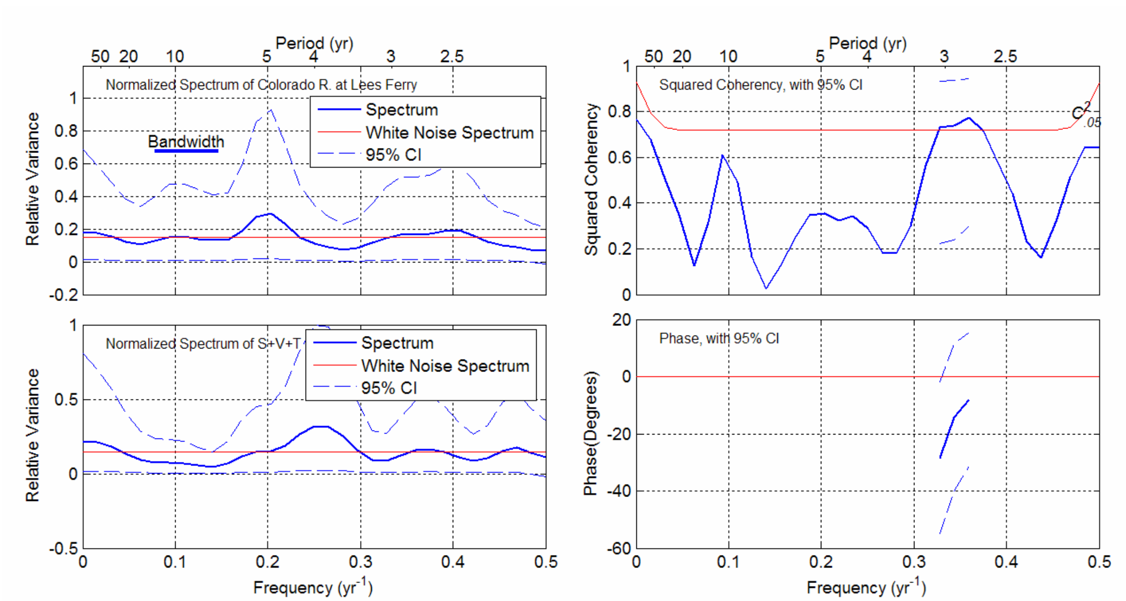


Figure 3. Cross-spectral analysis of reconstructed flows of Colorado River at Lees Ferry and Salt+Verde+Tonto River for calibration period, 1914-1964. Remainder of caption as in Figure 1.

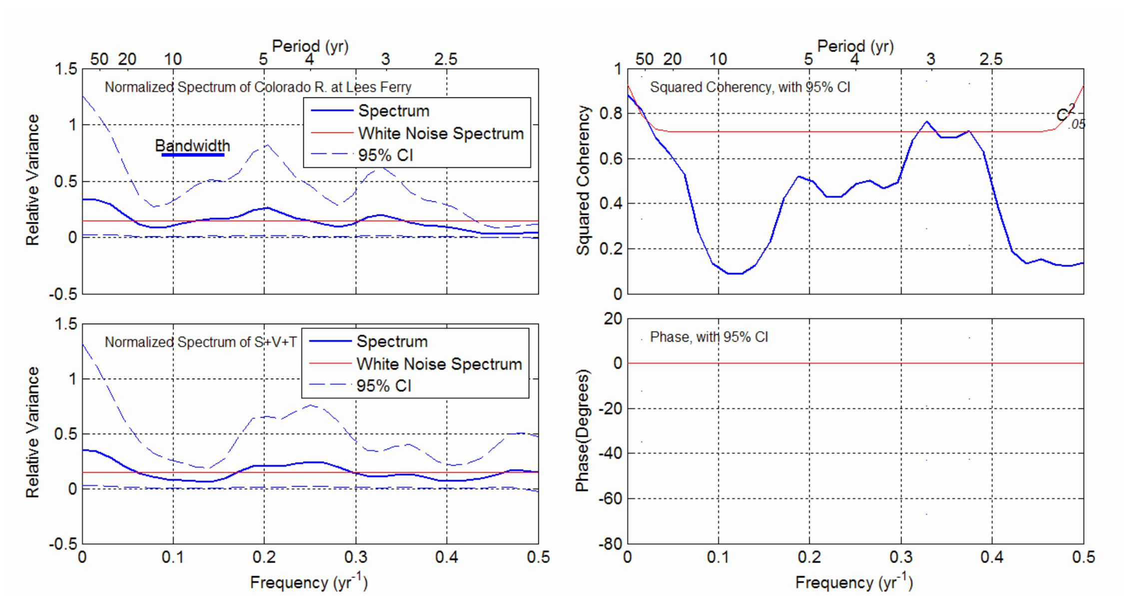


Figure 4. Cross-spectral analysis of observed flows of Colorado River at Lees Ferry and Salt+Verde+Tonto River for calibration period, 1914-1964. Remainder of caption as in Figure 1.