

ESTIMATION OF FLOW-DURATION CURVES AT UNGAGED SITES IN SOUTHERN NEW ENGLAND

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ABSTRACT

Two sets of regional-regression equations are developed to estimate the daily, unregulated, period-of-record flow duration curve (FDC) at ungaged sites in southern New England. The first method assumes an underlying probability density function (pdf) for daily streamflow whose parameter values are related to the physical characteristics of the ungaged basin. The second method relates flow at selected exceedence probabilities on the FDC to physical characteristics of the ungaged basin. We consider 66 relatively unregulated gages having between 10 and 86 years of continuous, daily-streamflow measurements. A jack-knife procedure is used to compare FDCs estimated from each method to the gage data from which the regression equations were developed.

FDC estimates from regression equations developed for individual exceedences led to lower mean square errors than estimates of FDCs that assumed an underlying pdf. L-moment diagrams, probability plots and simulation experiments reveal that daily streamflow are well approximated by a kappa distribution. The first four L-moments are highly correlated with each other, which were used to improve estimates of FDCs based on a regional kappa distribution.

INTRODUCTION

In Massachusetts, the legislation authorizes the Massachusetts Department of Environmental Protection (MassDEP) to assess permits of water withdrawals relative to the basin's safe yield, where safe yield is defined as the difference between the unimpacted streamflow at some location on a stream less some amount of water necessary to sustain the natural habitat. Estimates of unimpacted streamflow at any location on a stream – gaged or ungaged – is critical to the calculation of safe yield.

Previous work in Massachusetts to estimate streamflow at ungaged sites has employed regional regression to relate characteristics of an ungaged basin to selected flow-duration curve (FDC) statistics with the goal of estimating a daily, period-of-record FDC at the ungaged site. Ries and Friesz (2000) related physical characteristics of basins to selected exceedence probabilities associated with low flows. Flows at the 50-, 60-, 70-, 75-, 80-, 85-, 90-, 95-, 98-, and 99-percent exceedence probabilities were regressed against basin characteristics such as drainage area and percent of sand and gravel deposits in the basin. The resulting 10 equations provide a means to estimate streamflows less than or equal to the median flow at ungaged sites in Massachusetts. Alternatively, Fennessey (1994) and Fennessey and Vogel (1990) assumed an underlying probability density function for daily streamflow and regressed the parameters of the assumed distribution against basin characteristics. From L-moment diagrams (Hosking and Wallis, 1997) of daily streamflow, Fennessey (1994) argued that the three-parameter Generalized Pareto (GPA) distribution represents daily streamflow in New England. The resulting three regression equations were then used to obtain the daily, period-of-record FDC at an ungaged site.

MassDEP currently employs both the Ries and Friesz (2000) and Fennessey (1994) equations to estimate the partial or full FDC at ungaged sites for water management. Because two methods of estimating FDC statistics from regional regression are applied in Massachusetts, the purpose of this paper is to determine which of these two methods provides the best estimate of the daily period-of-record FDC. This study develops new regional regression equations because: (a) recent work by Castellarin et al. (2006) has suggested an alternative distribution to represent daily streamflow, and (b) higher resolution data sets of basin characteristics have been released since Ries and Friesz (2000) and Fennessey (1994) developed their regression equations.

For the purposes of this paper, we denote the estimate of a daily, period-of-record FDC curve from separate regressions at individual exceedence probabilities as the non-parametric FDC and the estimate of a daily, period-of-record FDC curve from an assumed probability distribution as the parametric FDC.

PROBABILITY DISTRIBUTION OF DAILY STREAMFLOW

Prior to the development of a parametric FDC, in the following section we explore the probability distribution of daily streamflow at the 66 minimally-impacted gages in southern New England shown in Figure 1.

Streamflow-gaging stations

The drainage basins to the 66 gages shown in Figure 1 do not contain major water withdrawals or return flows and the predominant land cover in the basin is forest; however, the presence of dams within the basins was not considered in the identification of minimally-impacted sites. Daily, continuous streamflow observations at the gages ranged from 10 to 86 years, with 47 gages containing greater than 20 years of continuous record and the drought of the 1960s, which is considered to be the drought-of-record in New England (fig. 1).

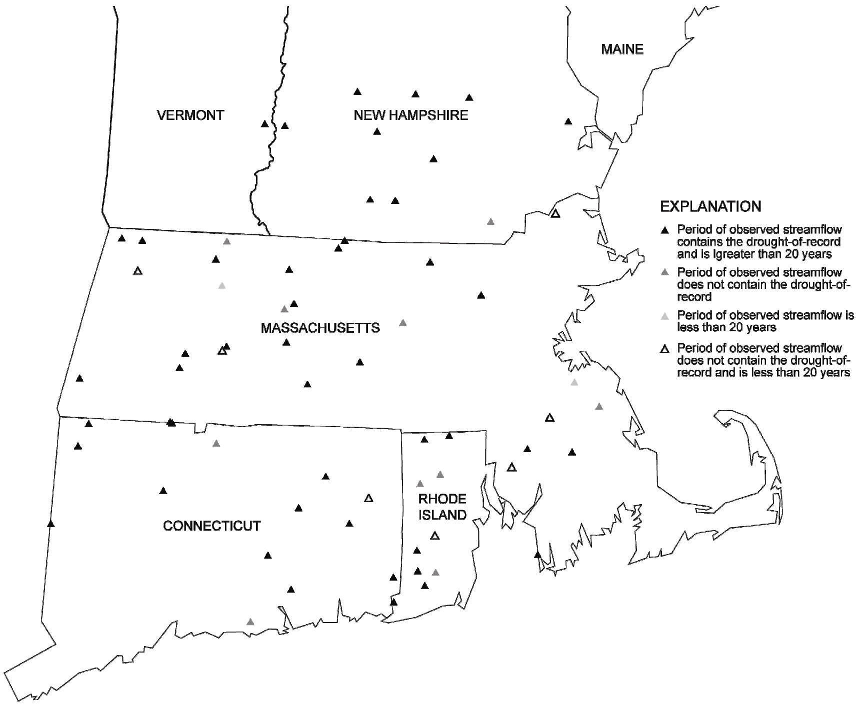


Figure 1. Locations of 66 minimally-impacted streamflow gages in southern New England.

L-moment diagrams

Prior studies by Vogel and Fennessey (1993, Figs 2-4) and Fennessey (1994) document that a generalized Pareto distribution provides a good approximation to the pdf of daily streamflows in Massachusetts and New England, respectively. L-moment diagrams comparing the sample L-moments to theoretical relations between L-moments for known probability distributions (Hosking and Wallis, 1997) are depicted. More recently, Castellarin et al. (2006) suggested that daily streamflow is better described by a four parameter Kappa (KAP) distribution for their estimates of period-of-record FDCs in Italy.

Sample L-moments were computed from the observed streamflows at the study gages and plotted on an L-moment diagram (fig. 2). The L-moment diagram includes common distributions used to represent various FDC statistics, including the GPA distribution, which Fennessey (1994) assumed to represent the distribution of daily streamflow in New England (fig 2). Also shown on the diagram is the kappa (KAP) distribution, which encompasses the area between the Generalized Logistic (GLO) distribution and the overall lower bound (OLB) on L-moments (fig. 2). The KAP distribution is a four-parameter distribution and includes the GPA distribution as a special case.

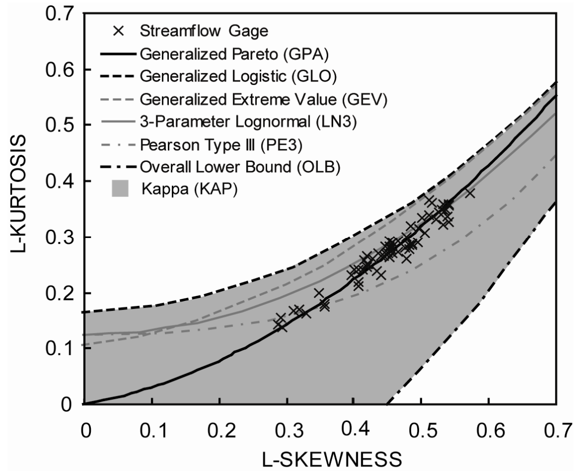


Figure 2. L-moment diagram showing sample L-moments computed from daily streamflows at 66 minimally-impacted gages and theoretical relations between L-kurtosis and L-skewness for selected probability distributions.

Visual inspection of figure 2 shows that the GPA distribution passes through the center of the sample L-moments with some scatter around the GPA line; however, because of the large sample sizes used to estimate the sample L-moments (n = 3,650 to 31,390 days), the scatter about the GPA line is not likely due to sampling. To test this hypothesis, parameters of GPA distribution were estimated from the sample L-moments for each of the 66 gages. The estimated parameters were then used to generate samples from a GPA distribution equaling the sample sizes at each of the respective gages. Sample L-moments computed from these simulated data sets were compared to the actual sample L-moments plotted in figure 3. If the underlying distribution of daily streamflows was GPA distributed, sample L-moments computed samples of sizes found at the study gages would plot as shown in figure 3b. From a comparison of figures 3a and 3b, it is apparent that the scatter about the GPA line is not due to sample size. We conclude the KAP distribution is a more appropriate representation of daily streamflow than the GPA distribution. The assumption that daily streamflows are KAP distributed is used to develop the parametric FDC estimate.

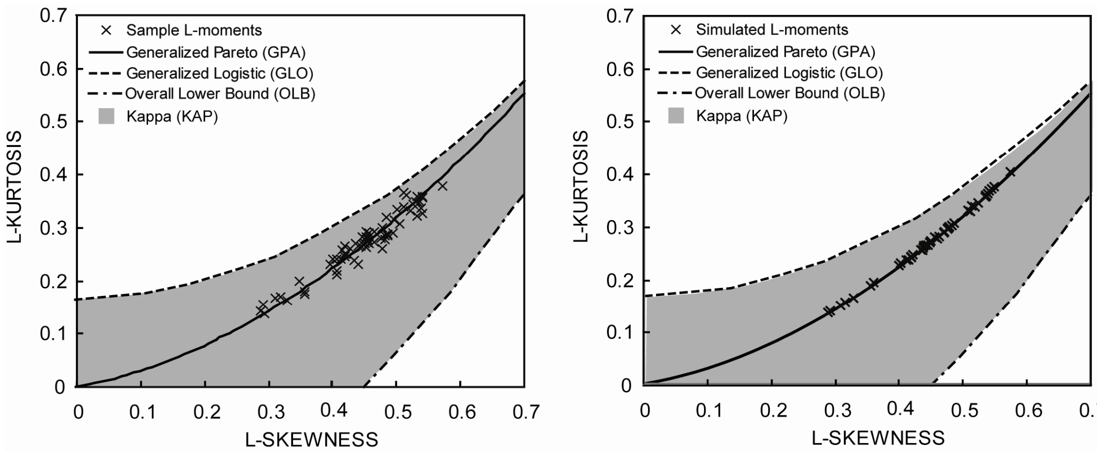


Figure 3. Comparison of (A) sample L-moments computed from daily streamflow at 66 minimally-impacted gages and (B) sample L-moments generated from a Generalized Pareto distribution.

DEVELOPMENT OF THE REGRESSION EQUATIONS

Selected basin characteristics and FDC statistics for each of the study gages were computed. Regression equations were developed from the initial set of 66 gages and a 47-gage subset of the 66 gages. The 47-gage subset contained gages that contained the drought of record and had greater than 20 years of observed streamflows. A consistent procedure was used to evaluate the validity of each regression equation developed in this study.

Basin characteristics

Basin characteristics – the independent variables in the regression equations – collected for the study gages are shown in table 1. The majority of the basin characteristics in table 1 were included in this study because they were found to be significant explanatory variables in previously-developed regressions equations at the study area. Other basin characteristics shown in table 1 were tested because they were easily computed or the basin characteristics are derived from recently released data sets not available for evaluation in previous studies' regression equations.

Table 1. List of basin characteristics computed for 66 minimally-impacted gages in southern New England.

Drainage area, in square miles
Mean basin elevation, in feet
Maximum basin elevation, in feet
Minimum basin elevation, in feet
Mean basin slope, as percent rise
Elevation at the gage, in feet
Mean annual precipiation, in inches
Average daily maximum temperature, in degrees Fahrenheit
Average daily minimum temperature, in degrees Fahrenheit
Location of the gage in the x-direction relative to Massachusetts State Plane coordinates, in meters
Location of the gage in the y-direction relative to Massachusetts State Plane coordinates, in meters
Percent of basin that is open water
Percent of basin that is wetlands
Percent of basin that is sand and gravel depoits
Percent of basin soil that is classificed as hydrologic soil group A
Percent of basin soil that is classificed as hydrologic soil group B
Percent of basin soil that is classificed as hydrologic soil group C
Percent of basin soil that is classificed as hydrologic soil group D
Percent of basin that is forested

Flow-duration-curve statistics

Flow-duration-curve statistics – the dependent variable in the regression equations – were estimated from observed streamflows at the study gages. The flow-duration curve statistics estimated are shown in table 2. To develop set of regressions that will estimate the non-parametric FDC, selected exceedence probabilities (table 2) were regressed against the basin characteristics listed in table 1. To develop set of regressions that will estimate the parametric FDC, KAP parameters were estimated using the routines described in Hosking and Wallis (1997) and regressed against the basin characteristics listed in table 1. In addition to estimating the parameters of the KAP distribution, moments, L-moments, and L-moment ratios (table 2) were also estimated and regressed against the basin characteristics. Moments, L-moments and L-moment ratios were estimated from the equations presented in Hosing and Wallis (1997). Because the KAP parameters themselves have little physical meaning, it was thought that the KAP parameters would not likely be related to physical properties of a basin; thereby resulting in poor regression equations. Alternatively, moments, L-moments and L-moment ratios do have physical meaning and would likely result in significant relations with basin characteristics. If the moments, L-moments and L-moment ratios resulted in better regression than the KAP parameters, the regression-estimated moments, L-moments, and L-moment ratios could be used to obtain the KAP parameters.

Table 2. Flow-duration-curve statistics estimated from 66 minimally-altered gages in southern New England

Exceedence probabilities		*Kappa Parameters*	*L-moment ratios*
1	60	α	λ_1
2	70	ξ	τ_2 (L-CV)
5	75	h	τ_3 (L-skewness)
10	80	k	τ_4 (L-kurtosis)
15	85		
20	90	***L-moments***	***Moments***
25	95	λ_1	μ (mean)
30	99	λ_2	s^2 (variance)
40	98	λ_3	m_3 (skew)
50		λ_4	k_4 (kurtosis)

Regression model and diagnostics

Ordinary least-squares regression was used to develop the regression equations and, with the exception of KAP parameters *h* and *k*, natural-log transformations were taken of the basin characteristics and flow-duration-curve statistics to linearize the relations between the two. Basin characteristics included in the final equation had variance-inflation factors less than 2.5. All model coefficients were significantly different from zero at the 0.05 significance level. Residuals (plotted in log space) were normally distributed with probability-plot-correlation coefficients greater than 0.93 and most residuals were normally distributed with greater than 75-pecent confidence. The R-squared-adjusted was close to the R-squared-predicted values and sites having large influence or a large standardized residual were removed. For regression equations developed in log space, bias correction factors were estimated by the Smearing Estimator (Duan, 1983) and applied to the final regression equations.

RESULTS

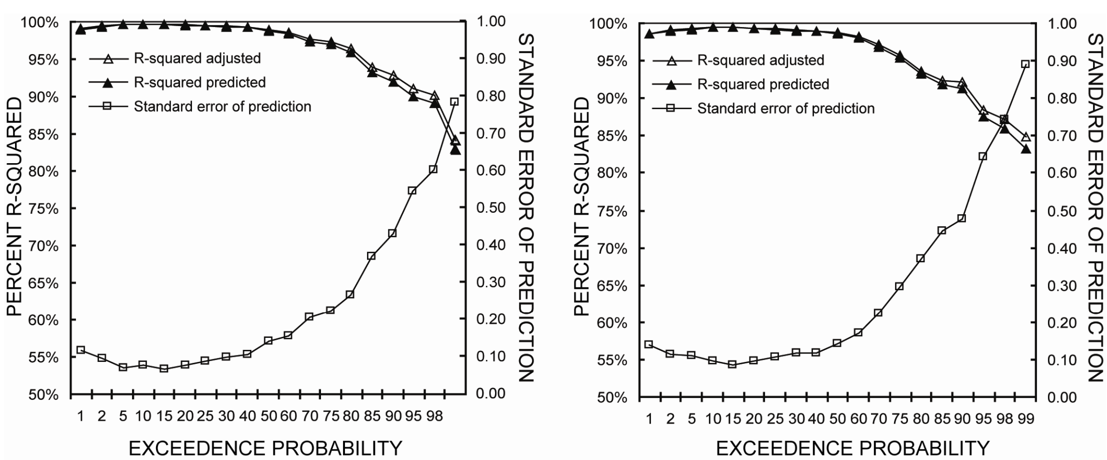
Regression equations were developed for the FDC statistics shown in table 2 from the basin characteristics listed in table 1. For each FDC statistic in table 2, two regression equations were developed: one from the 66-gage data set and one from the 47-gage data set. Of the 19 basin characteristics tested, 16 were found to be significant in at least one regression equation. Drainage area was found to be significant in all but the KAP-parameter equations, which supports the hypothesis that the KAP parameters have little physical meaning. Average annual precipitation, the location of the gage in the y-direction, and the percent of the basin that is wetlands are other basin characteristics that most-frequently occurred in the regression equations.

To quantify the differences associated with each FDC-estimation method, sites were jack-knifed. One site – the jack-knifed site – was left out and the regression coefficients were re-estimated. The FDC statistic for the jack-knifed site was computed from this new regression and compared to the observed value. The Nash-Sutcliffe efficiency value (Nash and Sutcliffe, 1970) was used to determine each method's utility in representing the period-of-record FDC at an ungaged site for each of the exceedence probabilities shown in table 2.

Regression equations for the non-parametric FDC

R-squared adjusted, R-squared predicted, and standard error of the regressions are shown in table 3. Generally, the equations have high R-squared-adjusted and R-squared-predicted values (greater than 90 percent). Note the decreasing trend in R-squared values and increasing trend in standard errors as the exceedence probability increases (table 3). This trend is consistent with the previous regression equations of low-flow durations in Massachusetts (Ries and Friesz, 2000). The regression equations for the high and mid-range exceedence probabilities contain only drainage area and location of the gage in the y-direction as explanatory variables; the regression equations for the high exceedences contain basin characteristics indicative of the capacity of the basin to store water, such as percent wetlands, percent water bodies, or soil characteristics.

Figure 4. R-squared adjusted, R-squared predicted, and the standard error of prediction for non-parametric FDC regression equations developed for flows at selected exceedences from (a) 47 and (b) 66 minimally-impacted gages in southern New England.



Regression equations for the parametric FDC

R-squared adjusted, R-squared predicted, and standard error for four sets of regression equations that determine the KAP distribution parameters – the KAP parameters themselves, moments, L-moments, and L-moment ratios – are shown in table 3. As expected, poor regression equations resulted when the KAP parameters were regressed against basin characteristics (table 3). Furthermore, the set of regression equations developed from the L-moment ratios did not result in the best regressions (table 3). Instead, moments and L-moments regressed against basin characteristics resulted in the highest R-square values and lowest standard errors (table 3). The set of L-moment equations were selected to obtain the KAP parameters because methods to transform L-moments to KAP parameters are better defined than methods to transform moments to KAP parameters.

Table 3. R-squared adjusted, R-squared predicted, and the standard error of prediction for sets of regression equations developed from 66 minimally-impacted gages in southern New England to estimate kappa-distribution parameters.

	Regressions developed from the 47-gage data set			Regressions developed from the 66-gage data set		
Kappa Parameters	*R-squared adjusted*	*R-squared predicted*	*Standard error of prediction*	*R-squared adjusted*	*R-squared predicted*	*Standard error of prediction*
α	99.20%	99.07%	0.11	98.70%	98.61%	0.13
ξ	57.60%	52.41%	13.54	41.40%	35.88%	12.47
h	72.10%	69.44%	0.15	56.90%	54.78%	0.18
k	71.30%	68.10%	0.06	66.90%	61.47%	0.28
L-moments						
λ_1	99.60%	99.53%	0.07	99.40%	99.35%	0.09
λ_2	99.50%	99.47%	0.08	99.30%	99.22%	0.10
λ_3	98.00%	97.62%	0.17	98.20%	97.88%	0.16
λ_4	97.60%	97.18%	0.18	97.10%	96.76%	0.20
L-moment ratios						
λ_1	99.60%	99.53%	0.07	99.40%	99.35%	0.09
τ_2 (L-CV)	60.00%	54.96%	0.07	51.80%	47.78%	0.07
τ_3 (L-skewness)	77.30%	72.17%	0.08	70.20%	66.51%	0.09
τ_4 (L-kurtosis)	81.10%	77.30%	0.11	66.00%	61.90%	0.14
Moments						
μ (mean)	99.60%	99.53%	0.07	99.40%	99.35%	0.09
s^2 (variance)	99.00%	98.78%	0.24	98.30%	98.20%	0.30
m_3 (skew)	97.60%	97.12%	0.54	97.00%	96.75%	0.59
k_4 (kurtosis)	94.00%	92.85%	1.07	95.20%	94.75%	1.00

In spite of the fact that Table 3 suggests that regression estimates of L-moments are highly accurate, the four regressions are estimated independently so that it is possible to obtain an infeasible combination of four estimated L-moments from the four independent regressions. Jack-knifed estimates of the parametric FDC regression estimates of L-moments at each gage were compared to the actual estimated L-moments (fig. 5) to understand why some infeasible KAP-parameter sets resulted from the jack-knife-estimated L-moments. Figure 5 indicates that the relation between L-skewness and L-kurtosis was not preserved when the four L-moment regression equations are used.

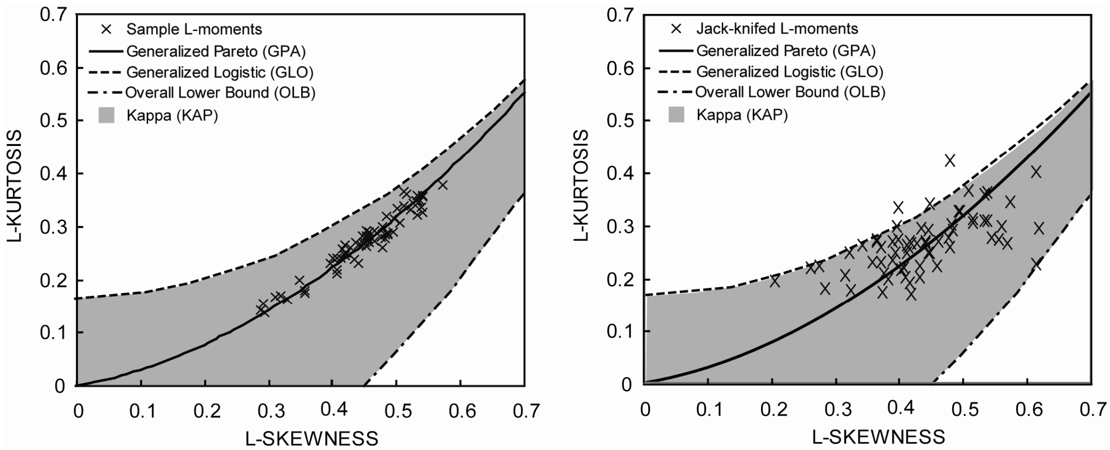


Figure 5. Comparison of (A) sample L-moments computed from daily streamflow at 66 minimally-impacted gages and (B) jack-knifed L-moments at 66 minimally-impacted gages computed from parametric regressions.

Plots of the L-moments versus one another revealed a very high cross correlation among the first four L-moments. There is no reason to suspect that the four independent regression equations of the L-moments (fig. 6) would preserve the correlation structure among the L-moments exhibited in Figure 6.

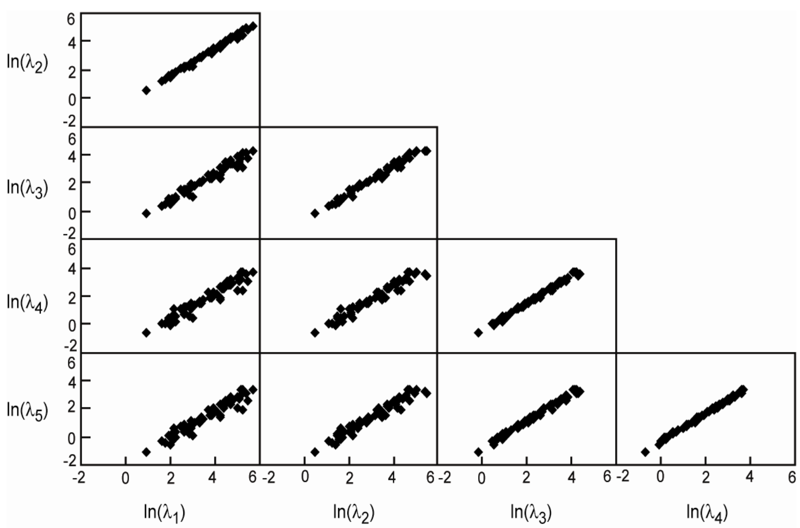


Figure 6. Relation between the natural logarithm of sample L-moments computed at 66 minimally-impacted gages in southern New England.

To improve the jack-knifed parameter estimates and preserve the cross-correlation structure of the L-moments, the relation between the L-moments shown in figure 6 was mimicked. The first L-moment λ_1 was determined from the regression equation. The procedure is as follows: The first L-moment is equivalent to the mean daily flow and the R-squared values were greater than 99 percent. Higher-order L-moments λ_2, λ_3 and λ_4 were determined recursively by fitting a linear relationship between λ_1 and λ_2, λ_2 and λ_3, and λ_3 and λ_4 (fig. 6). By preserving the empirical cross correlation structure of the L-moments, the jack-knifed L-moments resulted in feasible parameter sets that better represent the actual L-moment diagram (fig. 7).

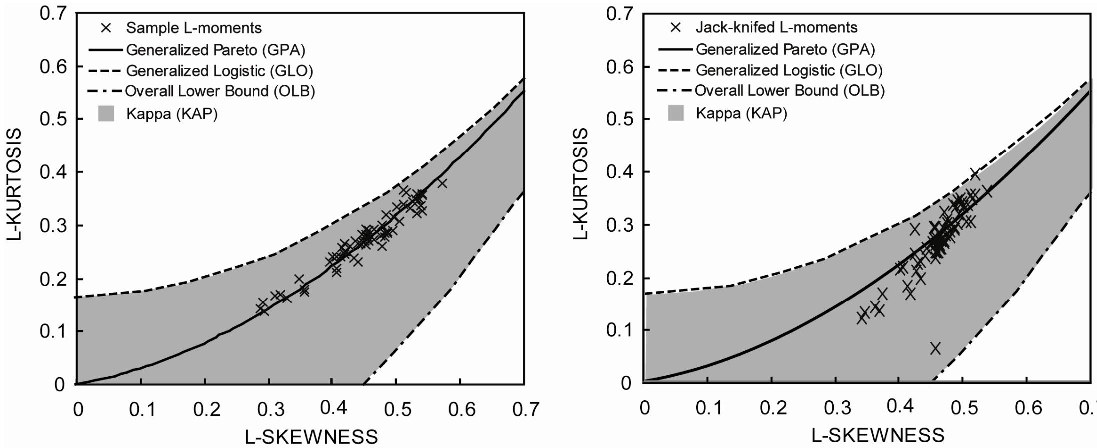


Figure 7. Comparison of (A) sample L-moments computed from daily streamflow at 66 minimally-impacted gages and (B) jack-knifed L-moments at 66 minimally-impacted gages computed from the recursive relations between the sample L-moments.

It is important to note that although the resulting KAP-parameter sets were feasible, some of the parameter sets returned negative streamflows because the values of these parameters resulted in theoretical lower bounds that were less than zero. In reality, streamflow values cannot be less than zero; therefore, for the cases where the parameter sets resulted in negative streamflows, the theoretical lower bound on the KAP distribution was set equal to zero. Though not applied to the analysis presented in this paper, Castellarin et al. (2006) shows an alternative way to constrain the lower bound of the KAP to avoid negative streamflows and preserve the first L-moment.

Comparison of methods

KAP parameters determined from the jack-knifed estimates of L-moments were used to estimate flows at the same 19 exceedence probabilities shown in table 2. The jack-knifed, parametric FDC estimates were then compared to the jack-knifed, non-parametric FDC estimates of flows at the same exceedence probabilities (fig. 8). Comparisons were made not only between the non-parametric and parametric FDC estimates but also between the 47- and 66-gage data sets (fig. 8). The Nash Sutcliffe efficiency value was used to compare the methods and data sets. Efficiency values of 1 indicate perfect agreement between simulated and observed values. Negative efficiency values indicate that the mean of the observed data is a better estimate of flows at a particular exceedence probability than the regression model itself.

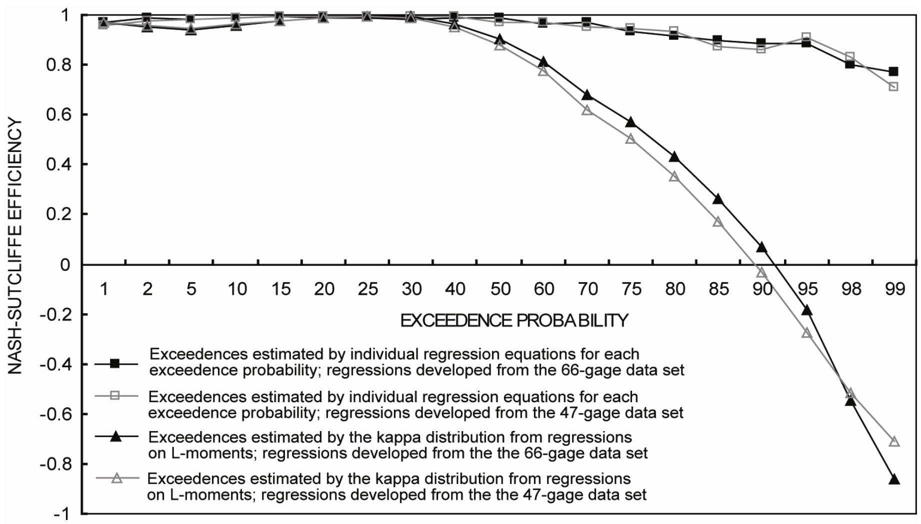


Figure 8. Comparison of non-parametric and parametric estimates of the daily, period-of-record flow-duration curves for selected exceedence probabilities based on Jack-knife simulation experiment.

It is clear from figure 8 that the non-parametric FDC estimates either perform as good as, or better than the parametric FDC estimates. For low exceedence probabilities (higher flows), the two methods perform similarly; however, at the high exceedences (lower flows), the parametric FDC estimates not only perform worse than the non-parametric FDC estimates, but for exceedences greater than 90 percent, the mean of the data is a better model than the parametric method (fig. 8). Alternatively, the efficiency values for the non-parametric estimates remain high (greater than 0.7) for flows at all exceedence probabilities (fig. 8). It also evident from figure 8 that there is no difference in efficiency values between the 47- and 66-gage data sets for either method. Therefore, including gages that had less than 20 years of observed streamflow or did not contain the drought of record did not appear to affect the efficiency values.

DISCUSSION

First, it should be noted that the substantial spatial correlation between observed streamflows at the study gages was ignored in this study. Weighted-least-square regression was used to account for the variations in record length at the study gages; however, in many cases the resulting regression equations had similar or worse standard errors and R-squared values, than when ordinary least squares regression was employed. It is also unclear how the nearly-perfect cross correlations among the various flows quantiles affected the non-parametric FDC regression estimates. No attempt was made to preserve the cross correlation among flows at various exceedence probabilities analogous to the attempt made to preserve the cross correlation among the L-moments which was exhibited in Figure 6.

It is not surprising that the set of regression equations for the non-parametric FDC estimate uniformly outperformed the parametric FDC regression estimates. The non-parametric method employs 19 separate equations to estimate the FDC, which allows for more curve-fitting – particularly at the highest and lowest exceedence probabilities – whereas the parametric method relies on four parameters to estimate flows at all exceedences on the FDC. The difficulty of the distribution to represent low streamflows was evidenced by probability plots of the observed and kappa-estimated quantiles. Although the non-parametric method is the clear method to choose for the goals of this study, it is unclear if future refinements to the assumption of the underlying distribution of daily flows or improvements in the estimation of the kappa parameters would yield a more competitive parametric method.

CONCLUSIONS

Two methods to estimate flow-duration curves (FDC) in Massachusetts were compared. The first method, which provides a parametric estimate of the daily, period-of-record FDC at an ungaged site, assumes an underlying probability density function for daily streamflow whose parameter values are related to the physical characteristics of the ungaged basin. The first method, which provides a non-parametric estimate of the daily, period-of-record FDC at an ungaged site, relates flow at individual exceedence probabilities on the FDC to physical characteristics of the ungaged basin. We consider 66 relatively unregulated gages having between 10 and 86 years of continuous, daily-streamflow measurements.

L-moment diagrams and simulation experiments led to the conclusion that daily streamflow in southern New England was kappa distributed. A jack-knife procedure compared FDCs estimated from each method to the gage data from which the regression equations were developed. Non-parametric FDC estimates definitively outperformed the parametric FDC estimates at the highest exceedence probabilities (the lowest streamflows). Regression equations at individual exceedences allowed for greater flexibility in estimating flows at the highest and lowest exceedence probabilities. L-moment diagrams and simulation experiments revealed that daily streamflow are well approximated by a Kappa distribution. Sample L-moments exhibit a very high degree of cross correlation and this fact was used to improve estimates of the parametric FDCs.

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