Tues, 3-12-19
Detrending

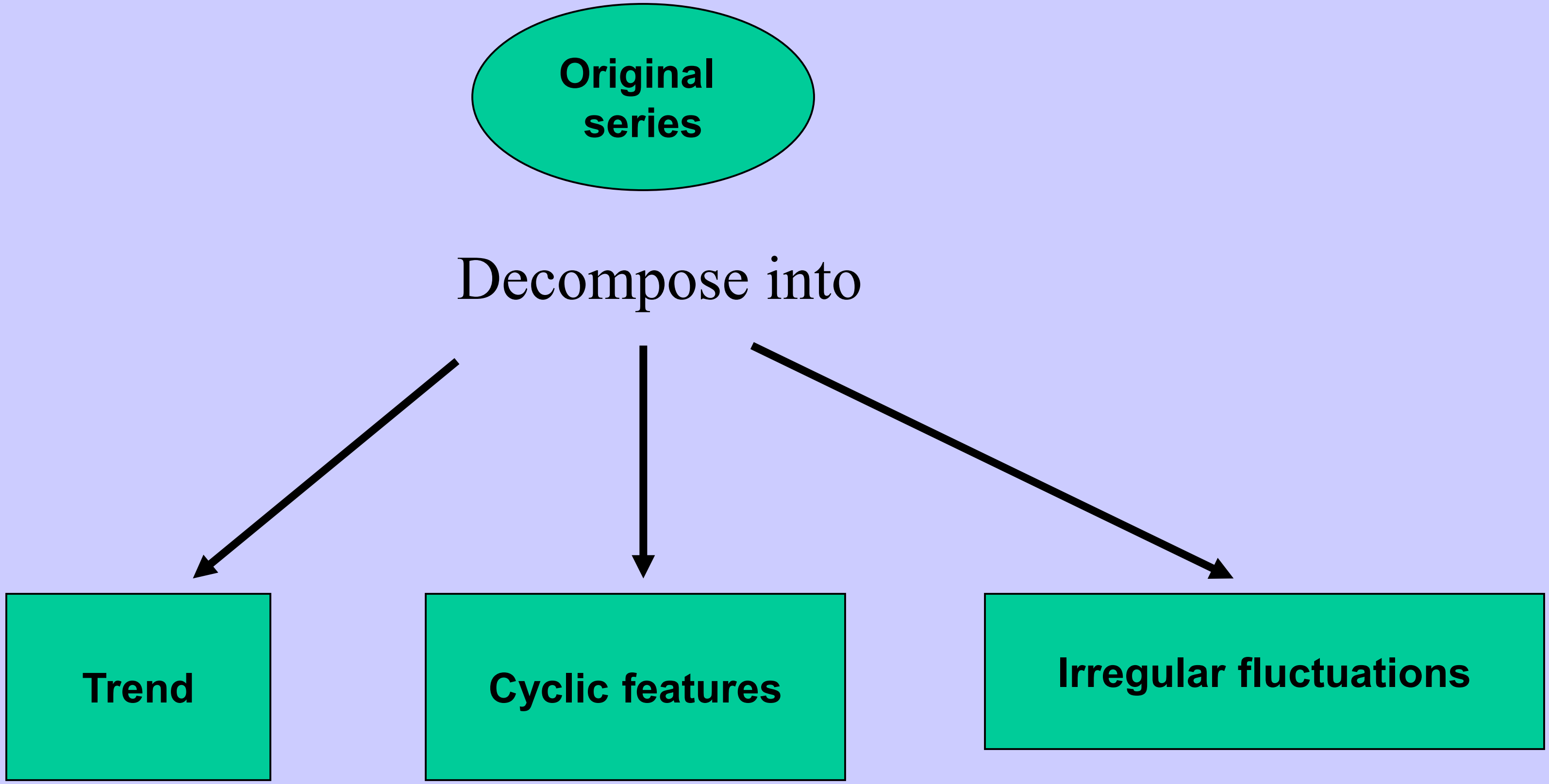
1. Lightning talk
2. Self assessment on A6
3. Identifying trend
4. Fitting and removing trend
5. Variance effects

Read notes_7.pdf

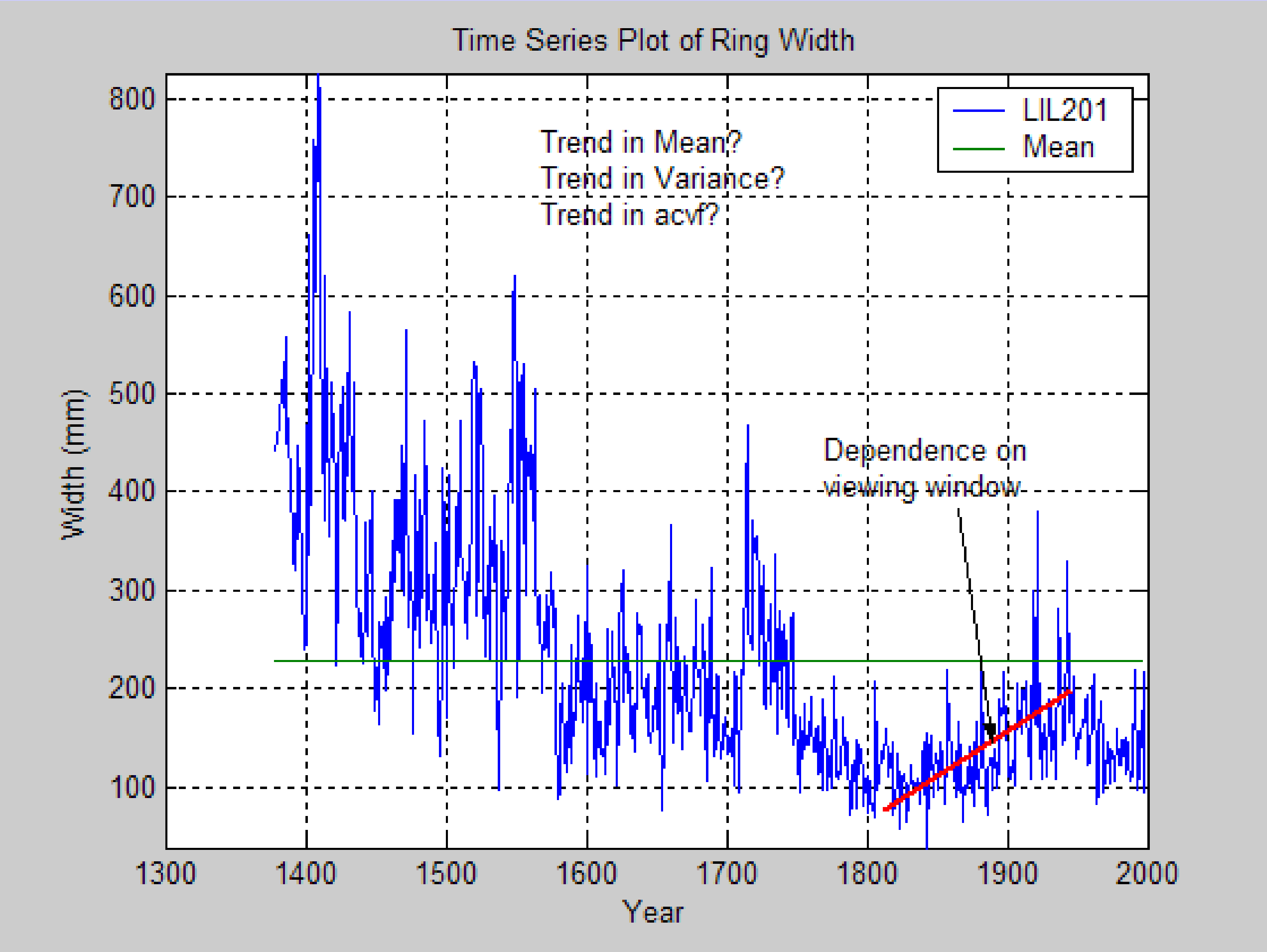
A6 Feedback

1. Download A6x.pdf from D2L
2. Automatic points, for running assignment and having uploaded by due time, is already marked in parentheses at top of first page
3. Each assignment has maximum possible 10 points; if you make no deductions, score is 10/10
4. A6x is color coded for points; purple=1; yellow=0.5; blue=0.5
5. Open your copy of the same assignment pdf you uploaded
6. In Acrobat Reader, using "Add text box," mark in right margin for deductions only, with deduction and segment reference : (eg., -0.5 A); round to tenths in deductions (e.g., no -0.25)
7. At top of your pdf, mark grade like this : 9.5/10
8. If necessary, put any comments at top near the grade
9. Upload your self-graded pdf to folder A5_graded in D2L

Traditional approach to time series analysis

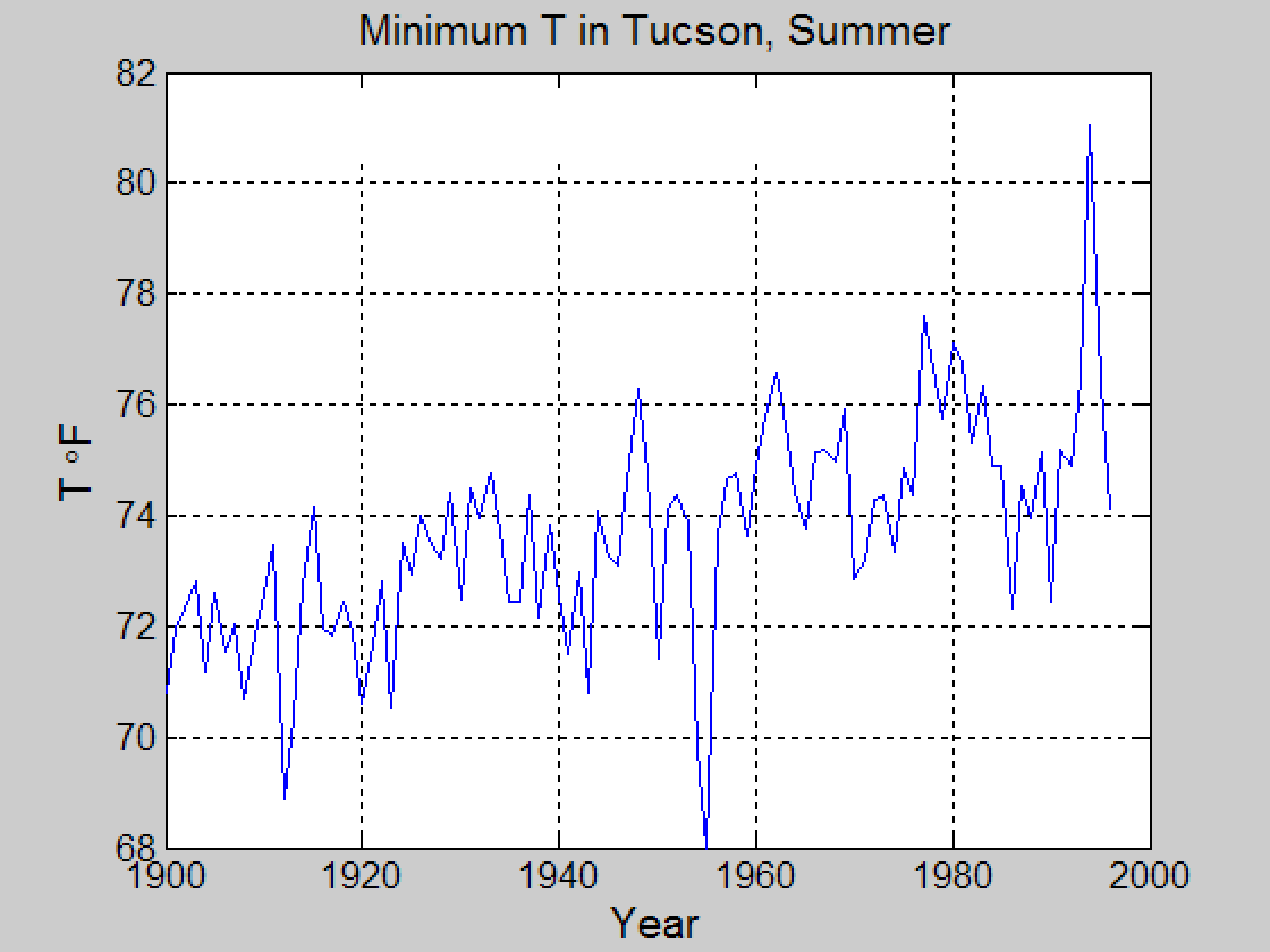


Identifying trend

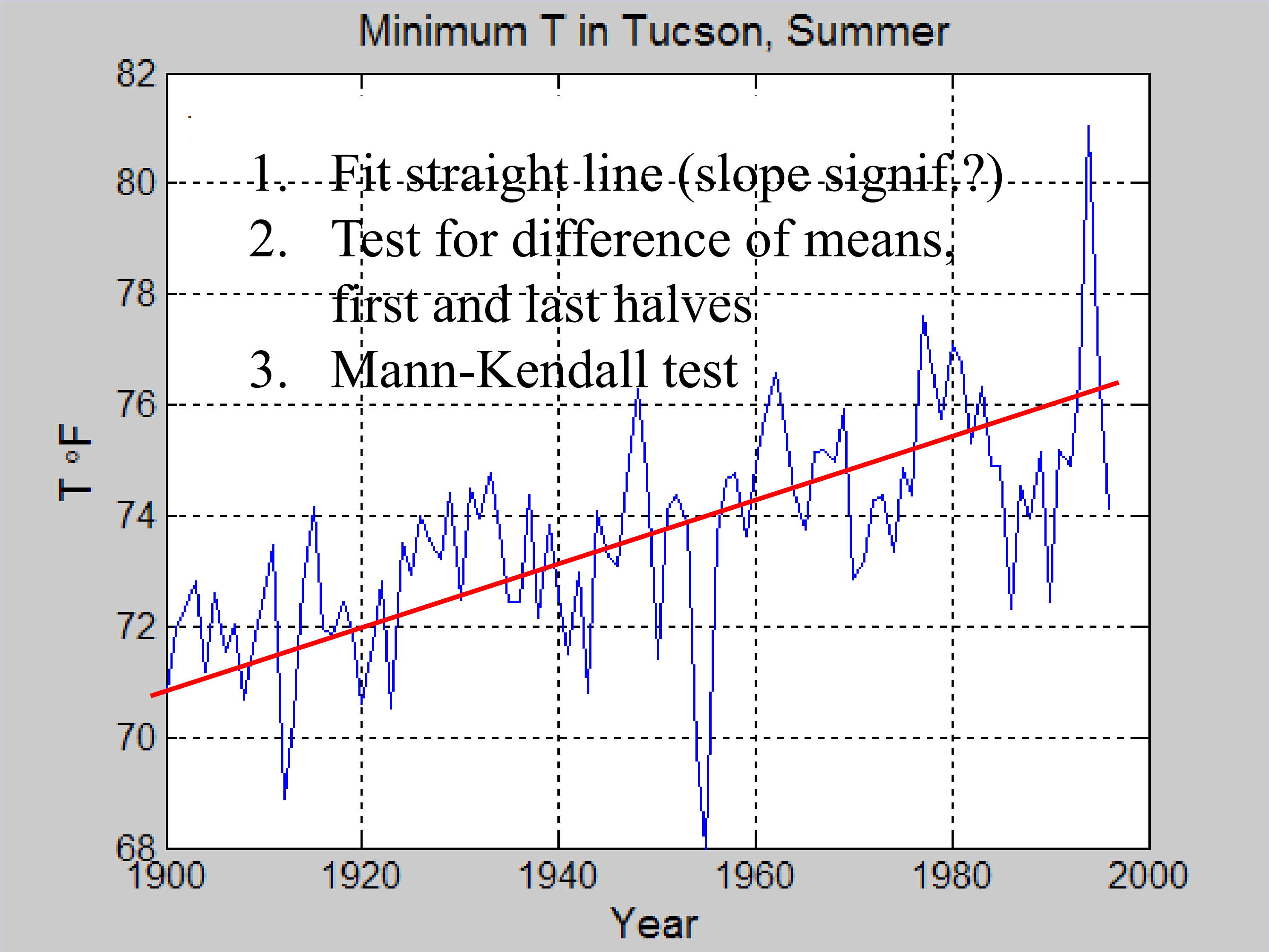


Detrending usually refers to trend in <u>mean</u>

Identifying trend--visually



Identifying trend -- statistically



Frequency-domain approach to identifying trend

$N =$ length of time series

$\lambda =$ wavelength of variation

Granger's* "trend in mean" rule (two versions):

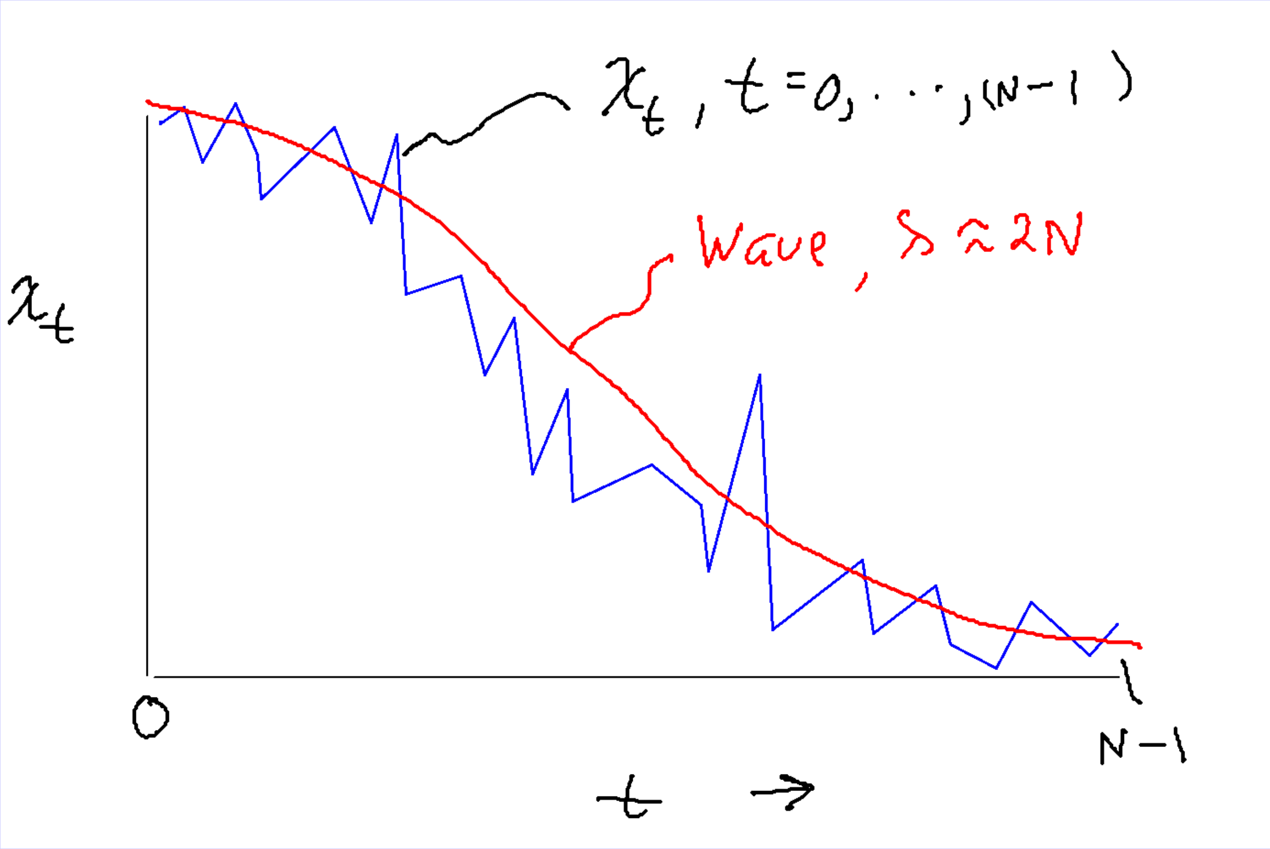
$1 -$ variations with $\lambda > 2N \rightarrow$ consider as TREND

$2 -$ variations with $\lambda > N \rightarrow$ consider as TREND

Variation (2) modified and adopted by Cook and Peters (see notes) for tree-ring detrending with cubic smoothing spline

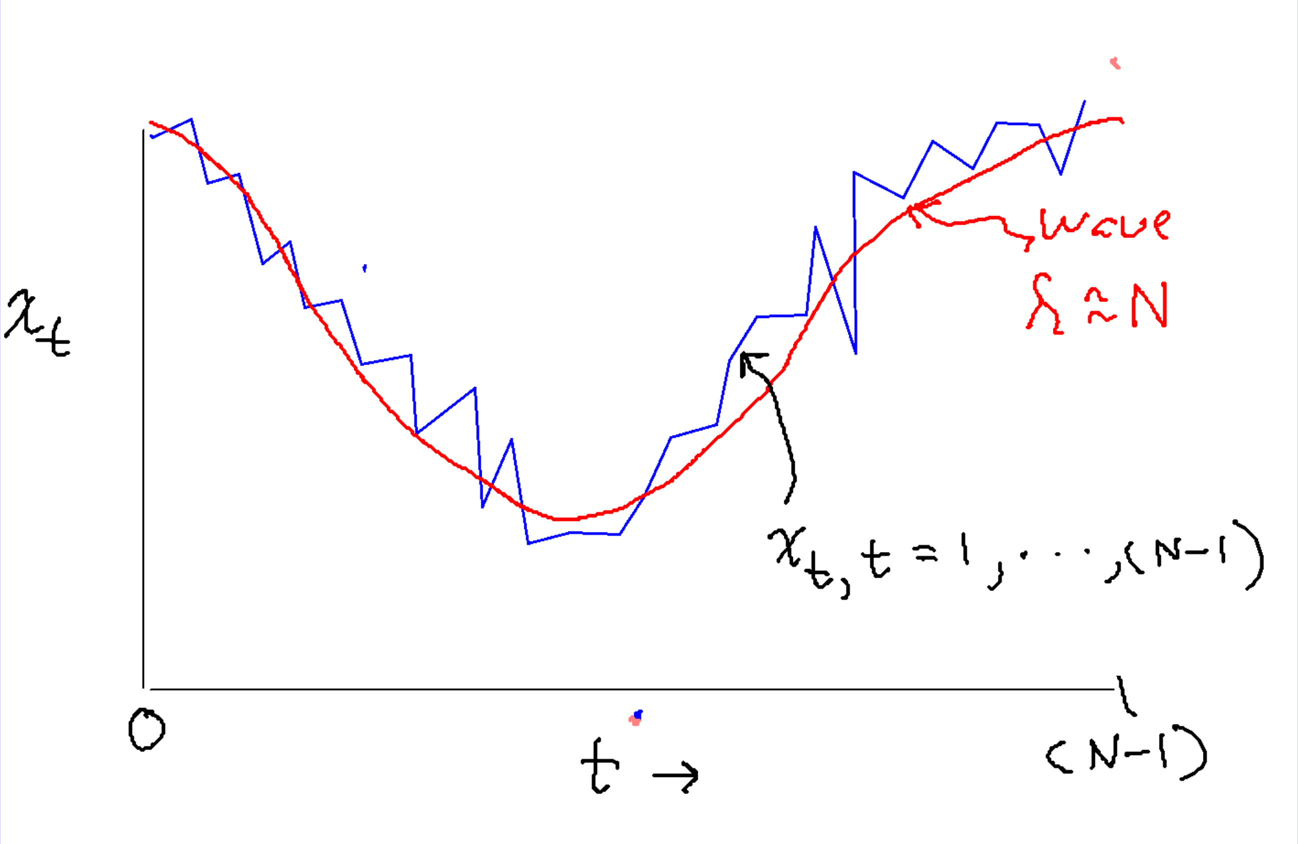
Granger & Hatanaka (1964)

“Trend in mean” (λ>2N)



- One full wave in 2N observations
- Remove it in detrending

“Trend in mean” (λ>N)



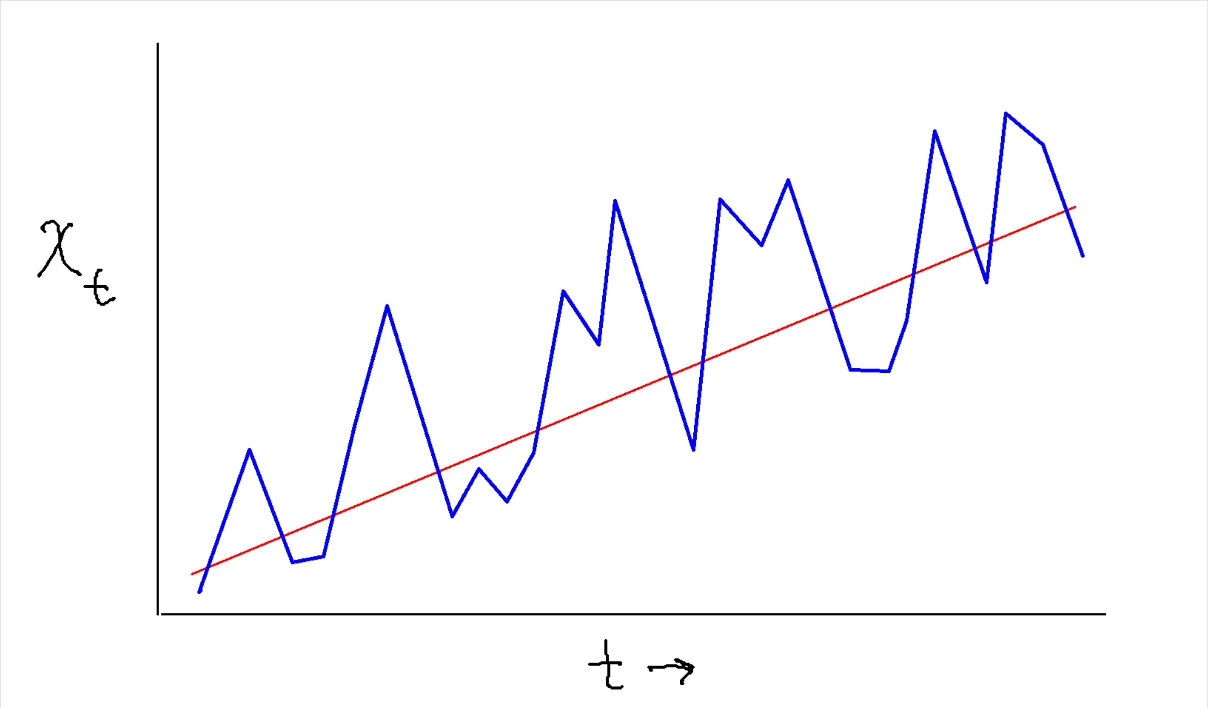
- One full wave in N observations
- Longest complete wave visible with length N series
- Remove it in detrending
- Cook & Peters (1981) used modified version of this rule iin guide for tree-ring detrending:

$$\lambda > \left(\frac{2}{3} \text{ to } \frac{3}{4} \right) N$$

Fitting the trend

1. Curve fitting
    - Global
    - Local
2. First differencing

Curve-fitting, global– linear trend

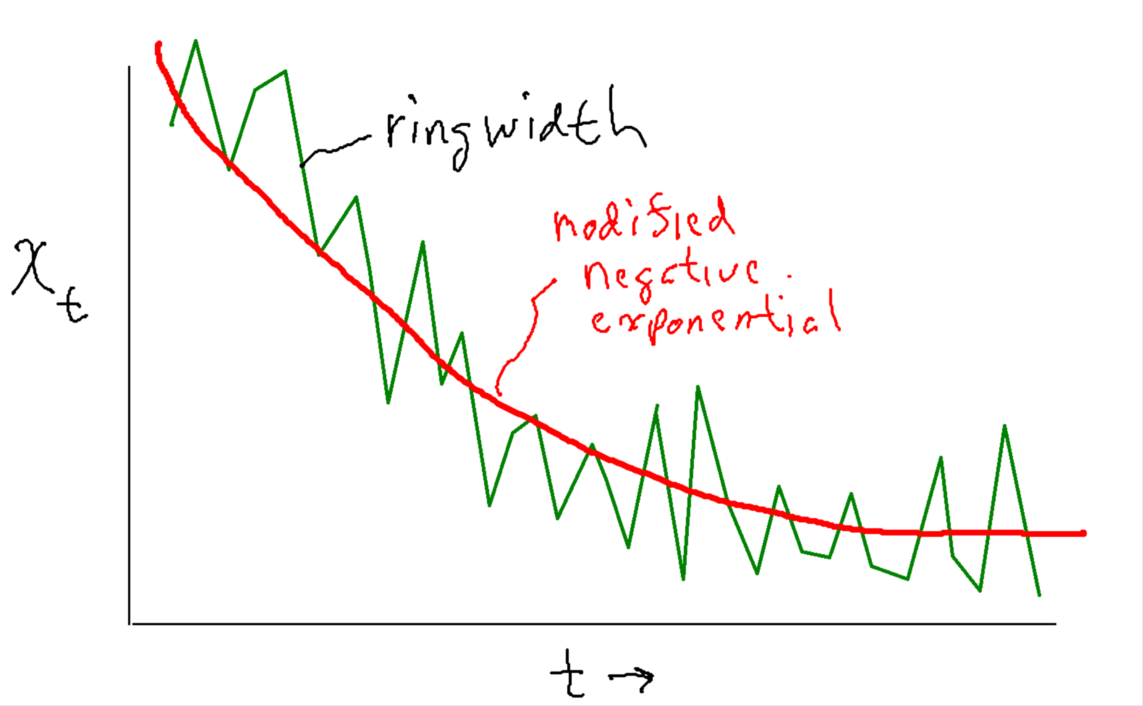


$$x_t = a + bt + e_t \quad \text{model}$$

$$g_t = \hat{a} + \hat{b}t \quad \text{fit}$$

Significance of estimated slope sometimes used as "test" for linear trend

Curve-fitting, global– nonlinear



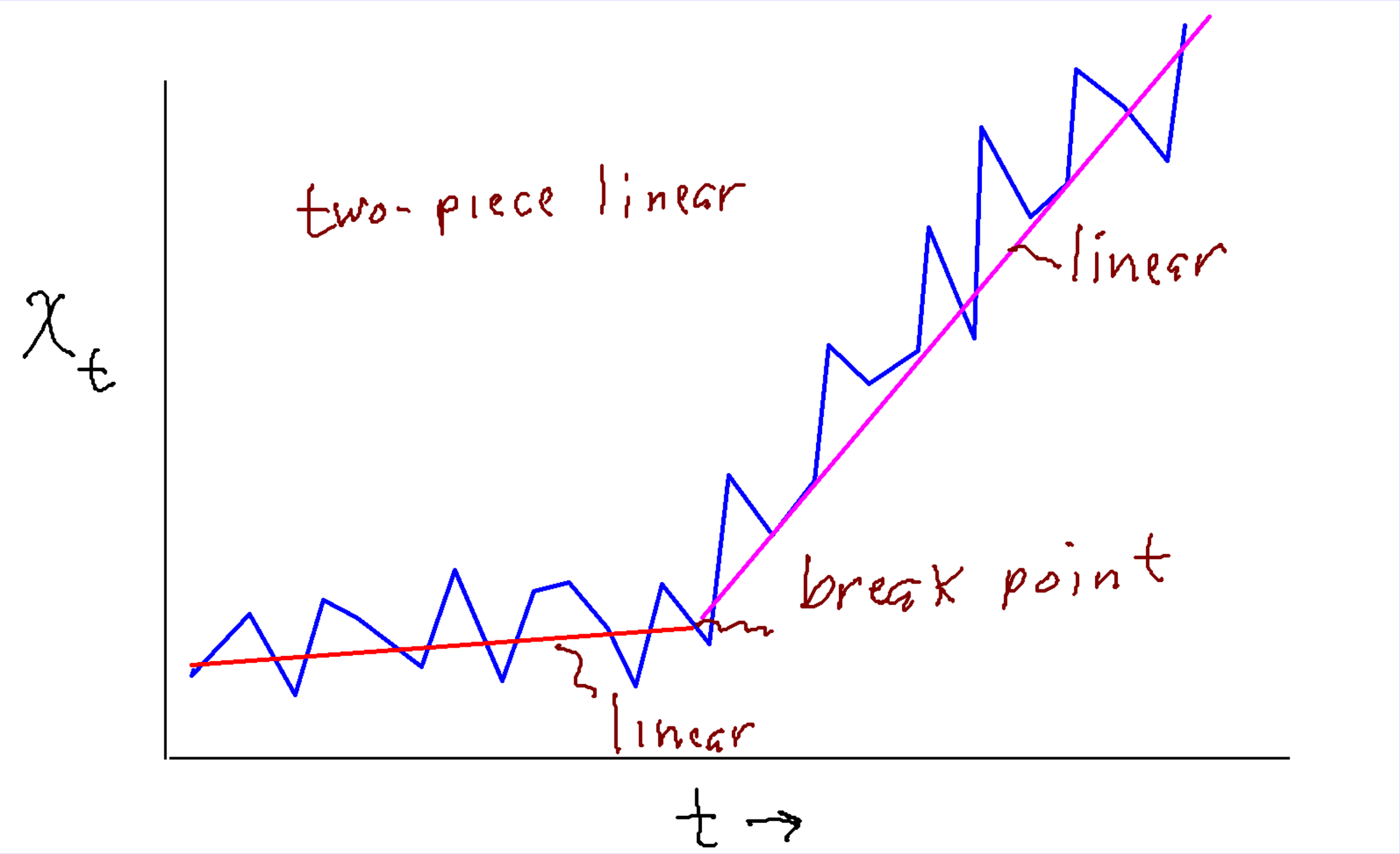
Tree-ring context

$$x_t = ae^{-bt} + k + e_t \quad \text{model}$$

$$x_t = \hat{a}e^{-\hat{b}t} + \hat{k} \quad \text{fit}$$

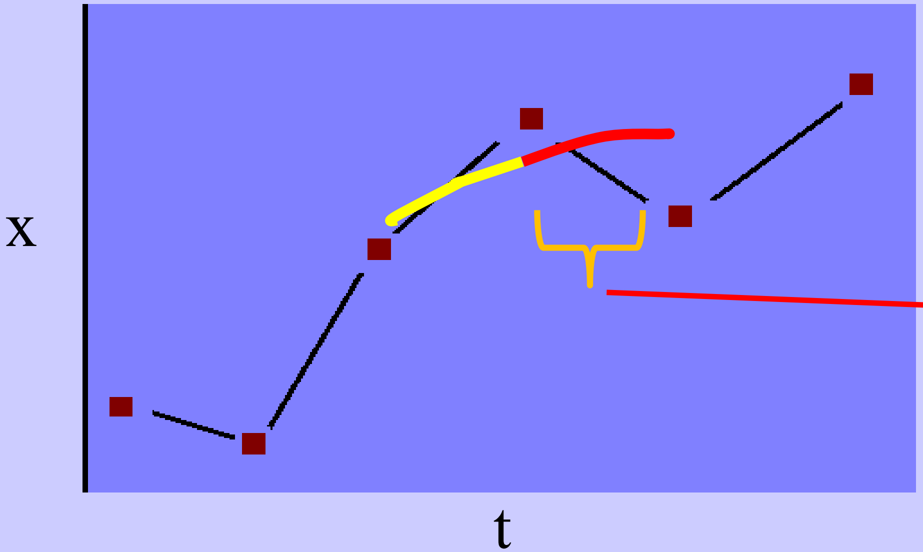
"modified" negative exponential, with limit k imposed on physical grounds

Curve-fitting, local – piecewise polynomial



e.g., with Matlab's
"detrend" function

Curve-fitting, local – cubic smoothing spline



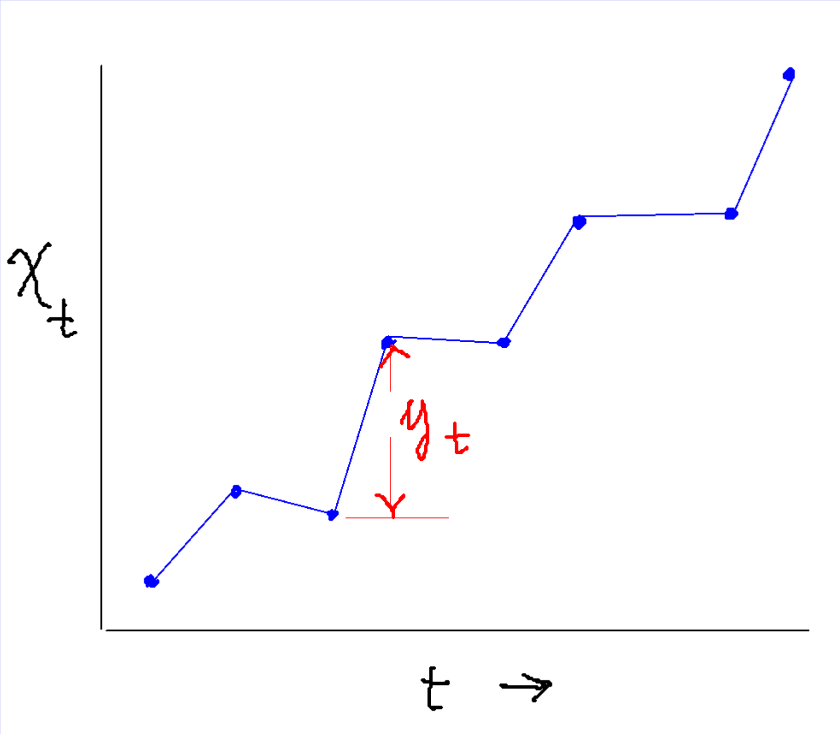
3rd order polynomial (cubic)

$$s_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Separate polynomial for each interval

1. Fit cubic polynomials locally
2. Each observation time is a "knot"
3. Estimate coefficients by solving system of equations with constraints: curves continuous at knots; D and D^2 continuous between at knots
4. Estimation differentially weights importance of
    1. Smoothness
    2. Closeness of fit

First differencing (and higher-order differencing)



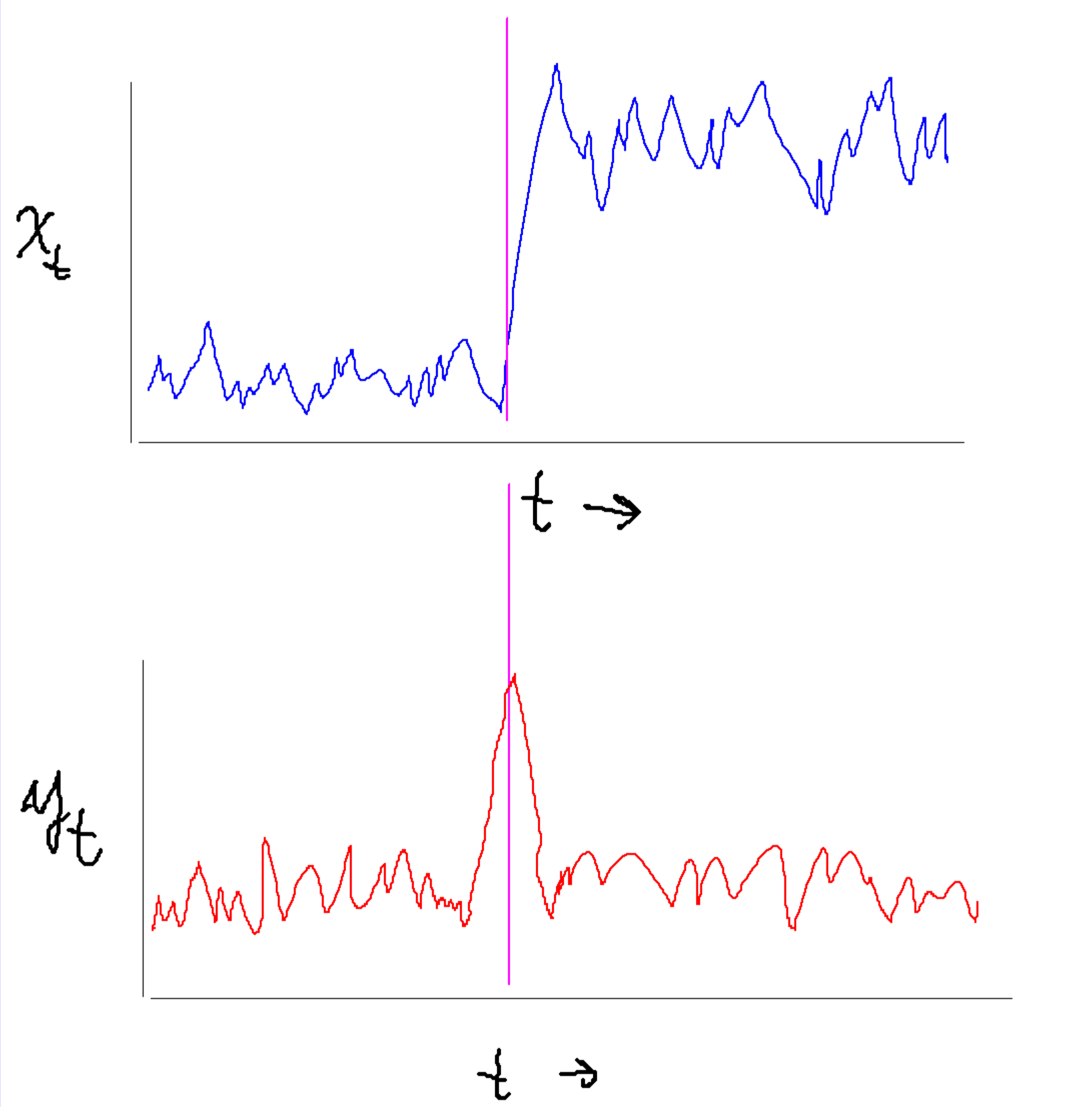
$x_t, t = 1, N$ time series

$y_t = x_t - x_{t-1}$ first difference

$w_t = y_t - y_{t-1}$ $2nd$ difference

- Used in stochastic modeling of trend (ARIMA models)
- 1st difference will remove linear trend
- Higher-order differencing might be needed for more complicated trend

First differencing with shift in level of time series



Change in level of x_t becomes spike in y_t

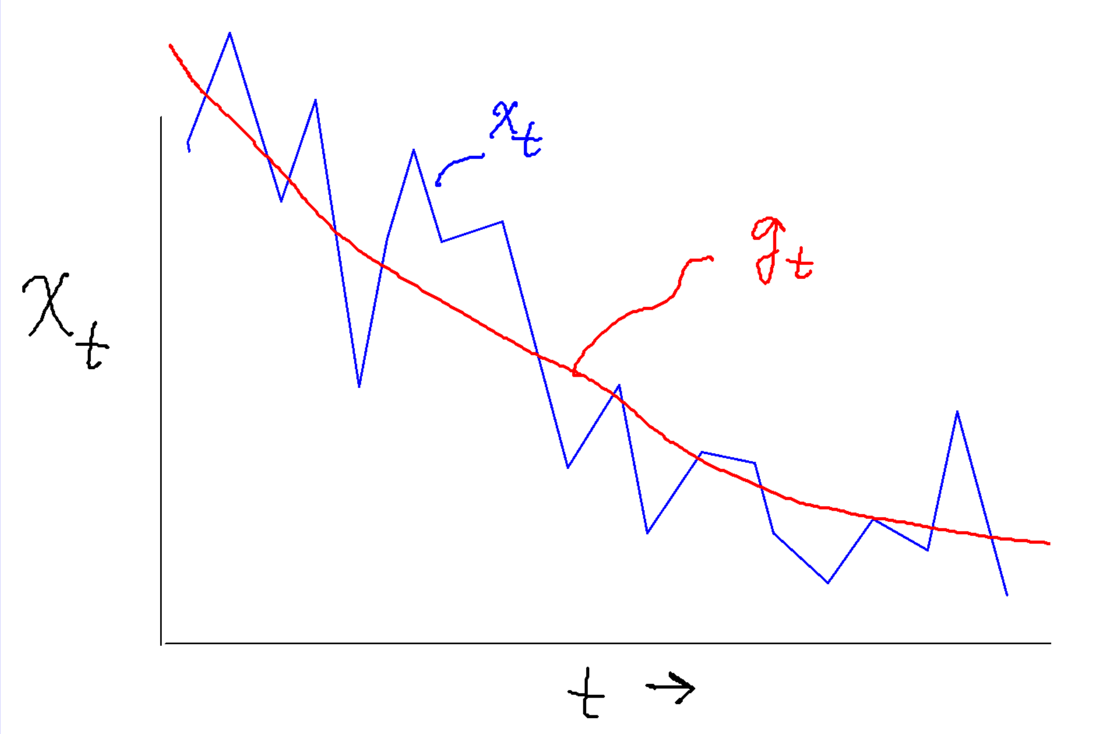
Removing the trend

1. Differencing: no action needed – differenced series ideally has the trend removed
2. Curve fitting
   - Difference method
   - Ratio method

Curve fitting: alternatives for removal of fitted trend

x_t original series

g_t fitted trend line



Difference

$y_t = x_t - g_t$

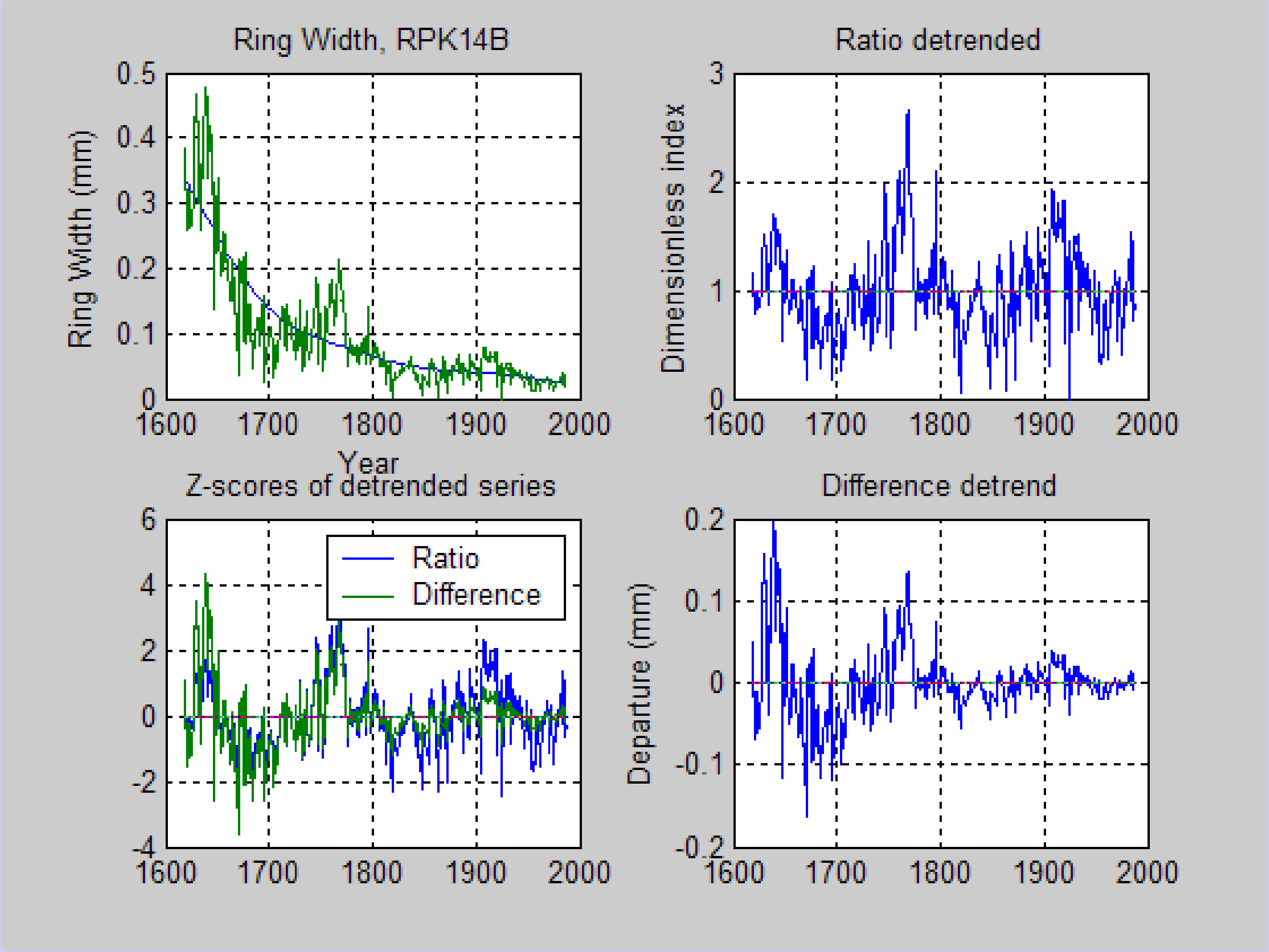
- -units unchanged
- -mean 0
- -variance neatly partitioned

Ratio

$$y_t = \frac{x_t}{g_t}$$

- -dimensionless
- -mean 1.0
- -variance not neatly partitioned
- -can also remove trend in variance
- -problems as $g \Rightarrow 0$

Example: Ratio vs Difference Detrending

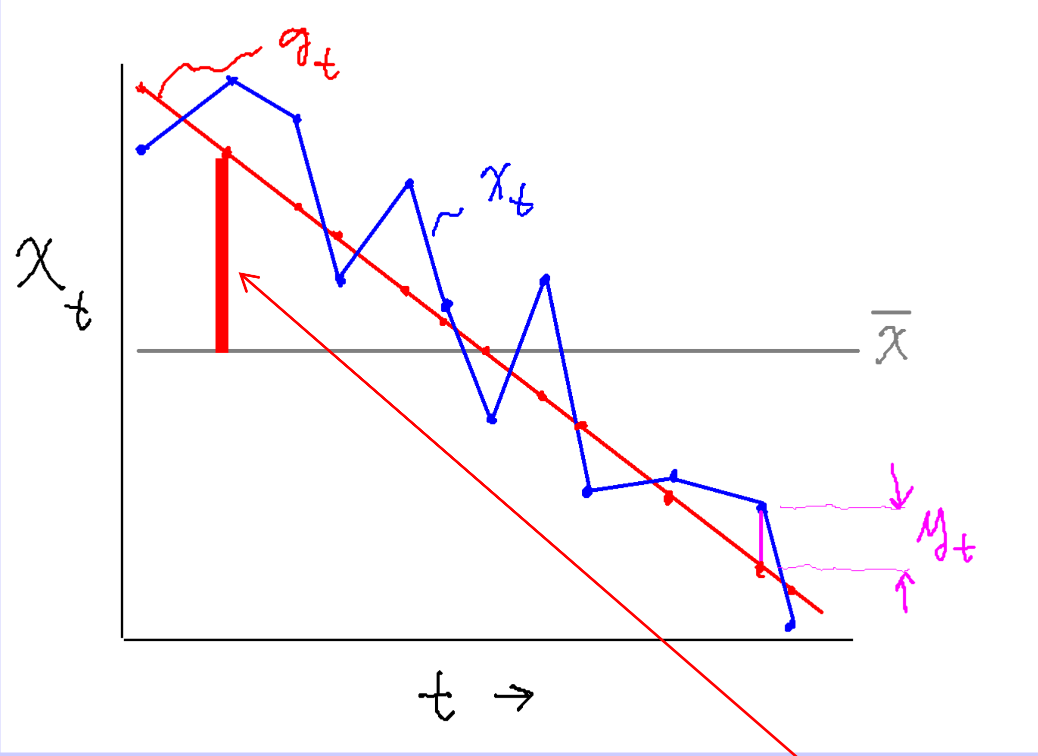


Effect of detrending on variance

Effect of detrending on the variance

x_t original series

g_t fitted trend line



$y_t = x_t - g_t$ detrended series

$\bar{y}_t = 0$

$$R_T^2 = 1 - \frac{\operatorname{var}(y_t)}{\operatorname{var}(x_t)}$$

Decimal fraction of variance due to trend

Variance of detrended series plus variance of trend line sums to variance of original series: this partitioning is true for differenc-detrending but NOT generally true for ratio-detrending

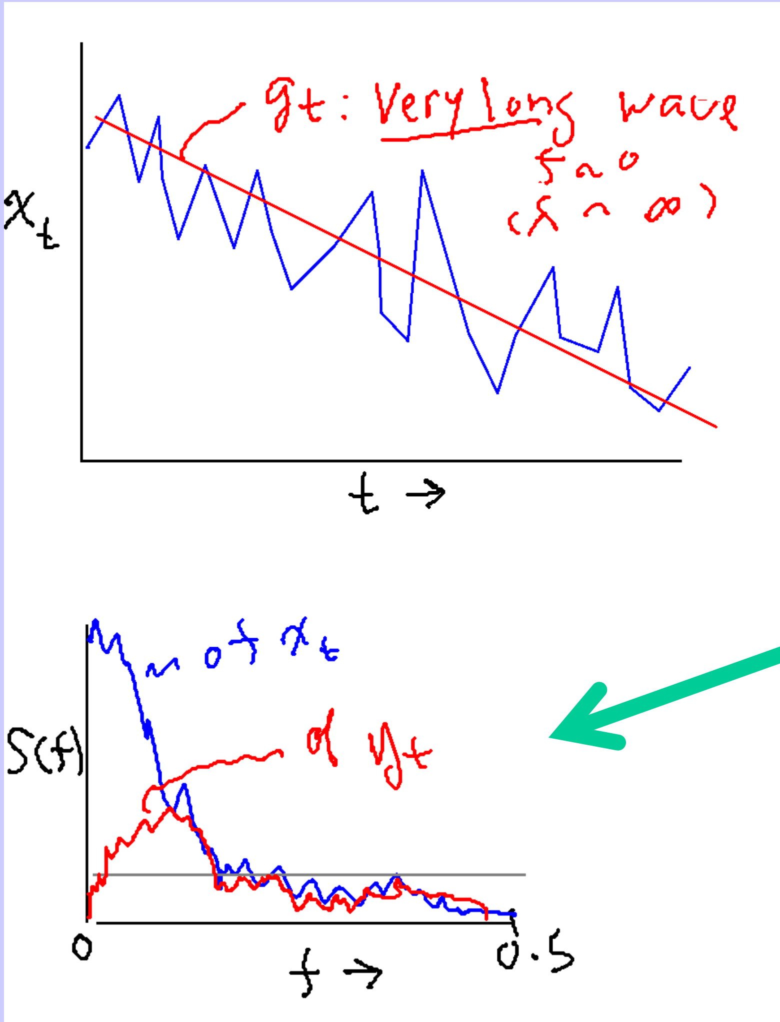
Departure of trend line from horizontal: the larger the mean square of these, the greater the variance due to trend

Effect of detrending on spectrum

- Fitted trend line → **low-frequency** feature
- Low-frequency variance proportion **less** in detrended series than in original series

Effect of detrending on the spectrum

e.g., linear trend



x_t original series

g_t fitted trend

$y_t = x_t - g_t$ detrended series

Variance is removed at lowest frequencies (longest wavelengths)