Thurs, 2-21-19

5. ARMA Modeling (cont.)

1. Checking the ARMA model
2. demo05a– simulating a time series with AR model
3. Sample runs of geosa5

Assignment a5: due Tues, Feb 26

Steps in Modeling

Structure and order?

1. **Identification**
2. **Estimation**
3. **Checking**

Values for parameters?

Are the residuals effectively without autocorrelation

Is the model that successfully removes the autocorrelation as simple as possible?

Checking

- The goal is to effectively model the persistence
- Questions
  1. Are the residuals white?
  2. Are the estimated model parameters significant?
  3. Can the model be simplified and still effectively whiten the time series?

The ARMA residuals

$y_t, t = 1, N$ the observed time series (as departures from mean)

This series is fit with an ARMA model, say AR(1)

$$\mathrm{y}_t + \hat{a}_1 y_{t-1} = \hat{e}_t$$

Residuals
Shocks
Whitened series

Are the residuals white (not autocorrelated)?

$\hat{e}(t)$ white?

Compute & plot acf of $\hat{e}(t)$

Evaluate $r_{\hat{e}}(k)$ at individual k

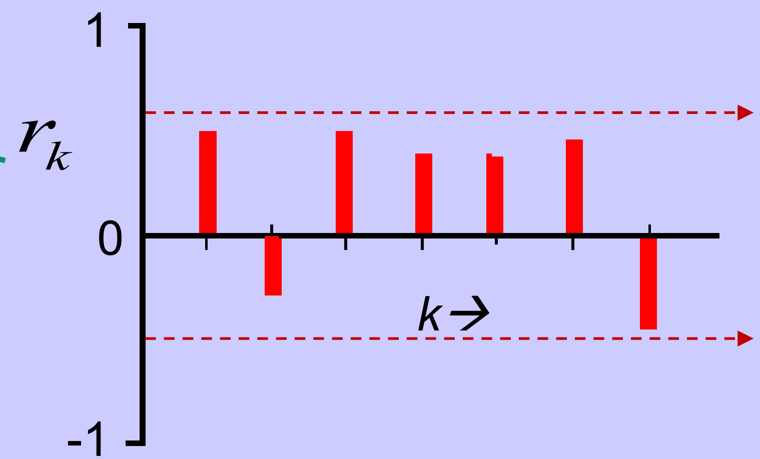
(as you did in assignment a3 for the time series itself)

Jointly evaluate set of K autocorrelations of $r_{\hat{e}}(k), k \leq K$

$K \approx 15$ to 30

"Portmanteau" or "Q" statistic

Portmanteau (Q) statistic



acf of $\hat{e}_t$ from ARMA model fit to time series $x_t, \quad t = 1, N$

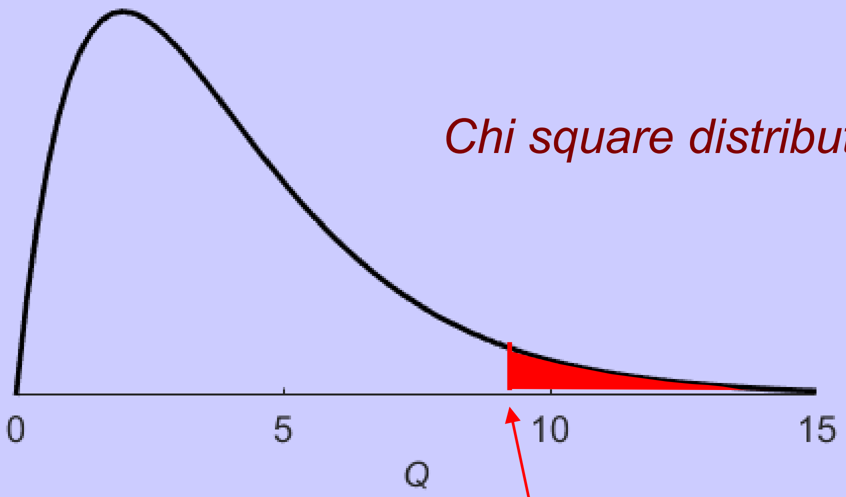
$r_{\hat{e}}(k)$

Simplified notation

$$Q = N \sum_{k=1}^{K} r_k^2 \quad \sim \chi^2 \text{ with df} = K - p - q$$

Typically K~ 15 to 30

Portmanteau (Q) lack-of-fit test



Chi square distribution

$$\alpha = 0.05$$

$H0:$ residuals white noise

$H1:$ residuals not white noise

$$|r_{\hat{e}}(k)| \quad \text{large} \rightarrow Q \text{ large} \rightarrow \text{reject H0 at } \alpha = 0.05$$

Caveat: test has "low power"

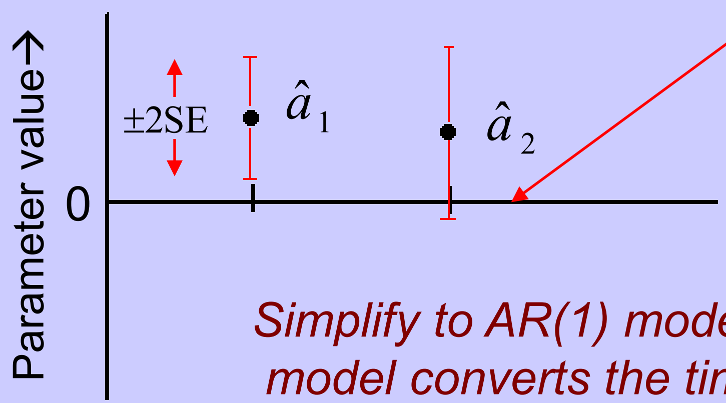
Is the model significant and simple?

Say, AR(2) model fit to data

$$y_t + \hat{a}_1 y_{t-1} + \hat{a}_2 y_{t-2} = \hat{e}_t$$

Are $\hat{a}_1$ and $\hat{a}_2$ significantly different from zero?

Can use the standard errors of the estimated parameters. These standard errors are functions of the estimated parameters, sample size and variance of the error term.



95% CI include 0? Estimated parameter not significant

*Simplify to AR(1) model, and check whether **that** model converts the time series to white noise.*

"parsimony" ***…*** *simplest model that does the job*

Variance due to (modeled) persistence

AR(1) model

departures of observed series from mean

$$y_t + \hat{a}_1 y_{t-1} = \hat{e}_t$$

$$v_p = 1 - \frac{\mathrm{var}\left(\hat{e}_t\right)}{\mathrm{var}\left(y_t\right)}, \qquad 0 < v_p < 1$$

- *Decimal proportion of variance due to modeled persistence*
- *Analogous to R^2 of regression*

Demo05a—example of simulation

1. Tree-ring index, Onion Creek, Calif, incense cedar
2. Use 1900-2014 part of series for modeling
3. Fit data to AR(1) model
4. Plot time series, its acf, and 5 simulations
5. Cut to command window for
   - Statistics of simulations
   - Acf plots of simulations

Trial runs of geosa5…