Thurs, 3-14-19

Detrending (cont.)

1. Cubic smoothing spline
2. Normalized spectrum
3. Demo07a: varying the spline parameter
4. Sample run of geosa7

Assignment a7: due next Tuesday

Cubic Smoothing Spline

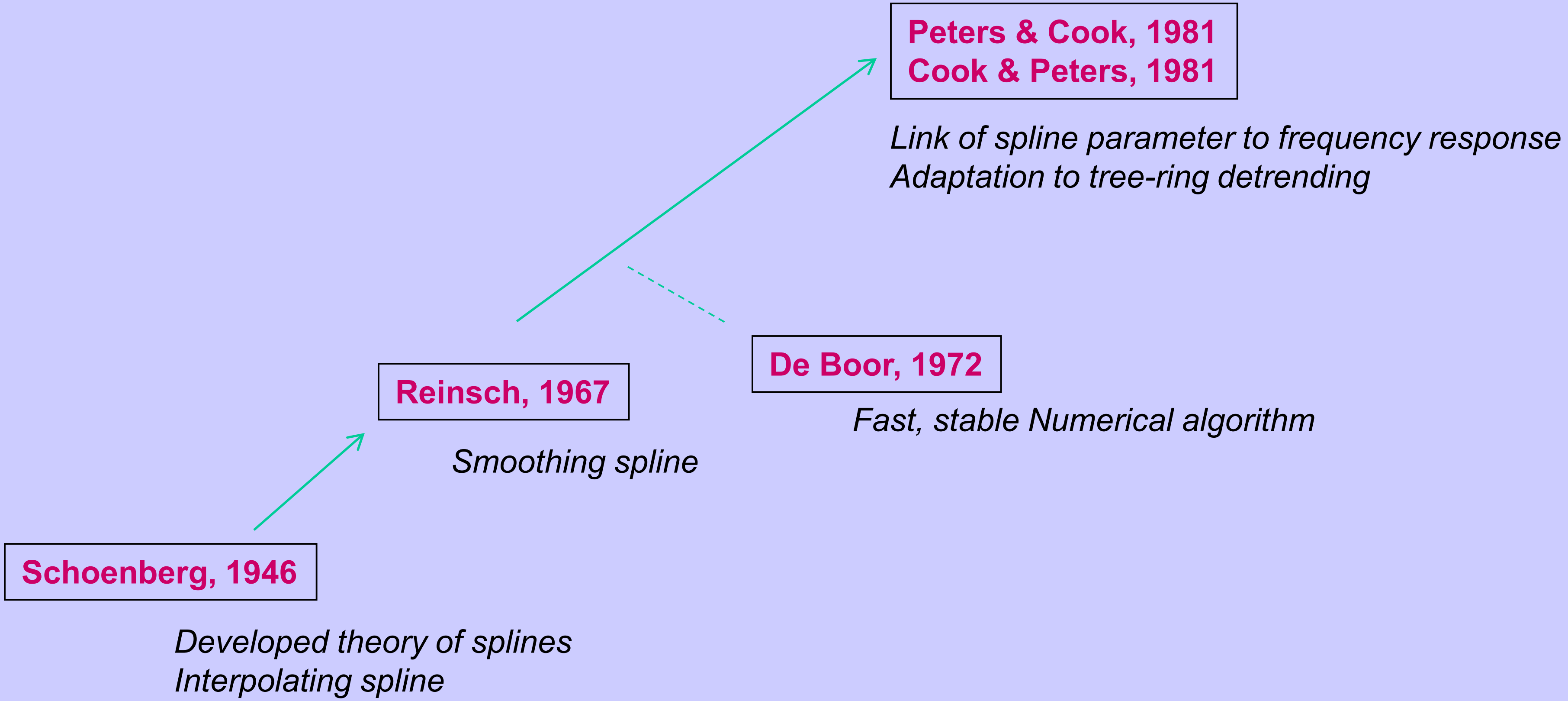
Cook, E.R., and Peters, K., 1981, The smoothing spline: A new approach to standardizing forest interior tree-ring width series for dendroclimatic studies,Tree-Ring Bulletin 41, 45-53.

de Boor, C., 1978, A practical guide to splines: New York, Springer-Verlag, 392 p.

Peters, K., and Cook, E.R., 1981, The cubic smoothing spline as a digital filter: Lamont-Doherty Geological Observatory of Columbia University, Technical Report no. CU-1-81/TR1: Palisades, New York, 21 p.

Reinsch, C.H., 1967, Smoothing by spline functions: Numerische Mathematik, v. 10, p. 177-183.

Schoenberg, I.J., 1946, Contributions to the problem of approximation of equidistant data by analytic functions: Quart. Appl. Math., v. 41, p. 45-99, 112-141.



Cubic Smoothing Spline

- Piecewise polynomial
- Highest order of polynomial is 3 : CUBIC
- “knots” at observation time in the series
- Control of flexibility with “spline parameter”: smoothness vs closeness of fit

Spline Parameter

The spline parameter minimizes the following quantity:

$$\overbrace{p\sum_{i=1}^{N}\left[\frac{y_i - s(x_i)}{\delta y_i}\right]^2}^{\textit{Closeness of fit}} + \overbrace{(1-p)\int_{x_1}^{x_N}\left[\mathrm{D}^2 s\right]^2}^{\textit{Smoothness}}$$

$p =$ spline parameter

$p = 1$ interpolant; all weighting on goodness of fit

$p = 0$ least squares straight line; all weighting on smoothness

$0 < p < 1$ somewhere between interpolating spline and straight line

$y_i =$ original time series, function of time $x_i, i = 1, \ldots N$

$s =$ spline curve, the smoothed version of y_i

$\mathrm{D}^2 =$ second derivative of s with respect to time

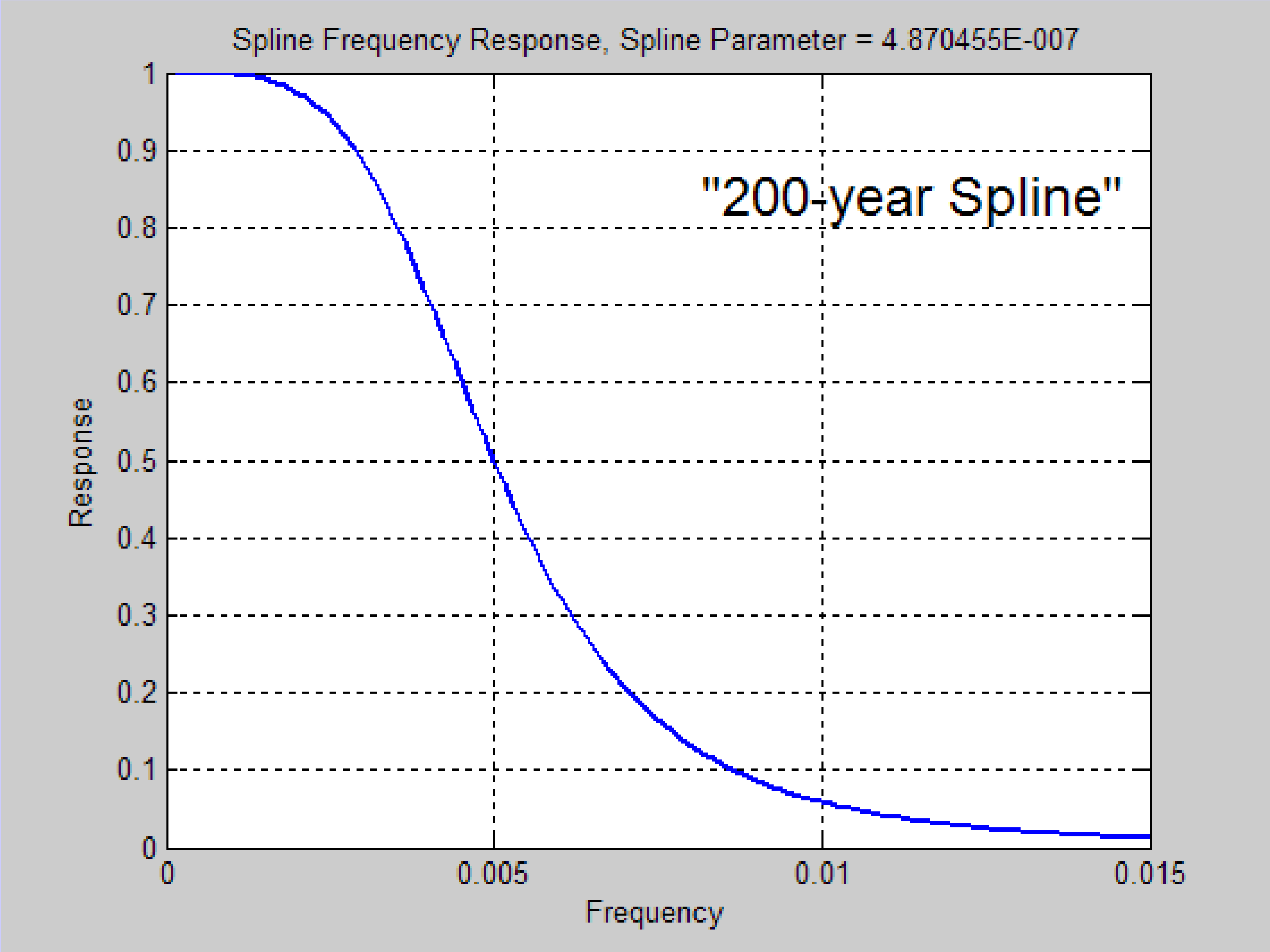
Frequency Response of Spline

- Describes how the amplitudes and phases of hypothetical sinusoidal fluctuations in the original series are tracked by the fitted smooth line
- Usually we are concerned with the <u>amplitude</u> of the frequency response
- The fitted trend line tracks low frequencies and ignores high frequencies: the amplitude of the frequency response is higher for the the low-frequency variations than for the high-frequency
- Plot of "frequency response function" shows variation of amplitude of frequency response as function of frequency

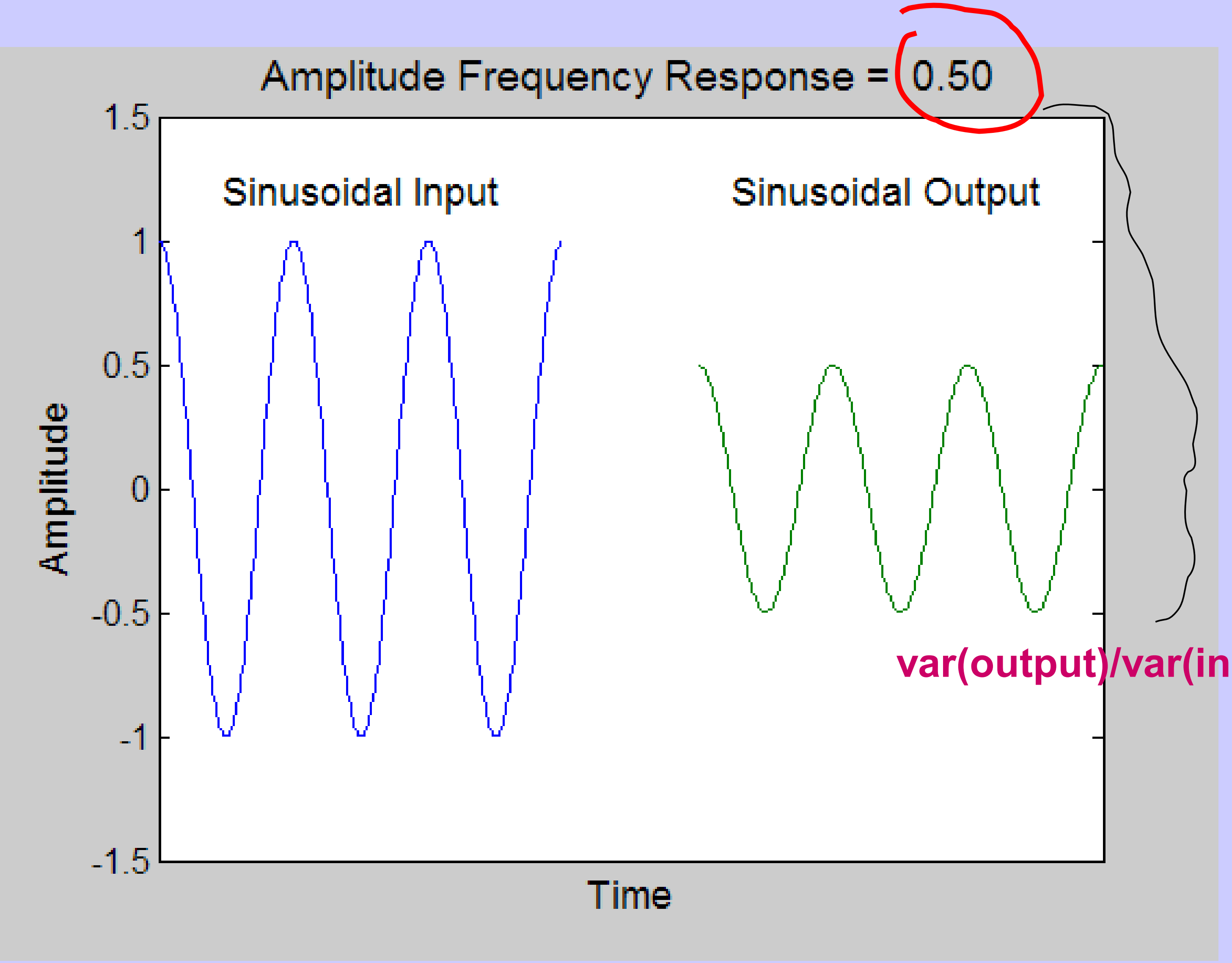
Tree-Ring Spline Terminology

- “N-year spline”: frequency response is 0.5 at wavelength of N years;
  - longer wavelengths tracked better, shorter not as well
  - those wavelengths removed when spline used in detrending
- “x%N spline”: frequency response is 0.50 at wavelength x percent of the length of the time series (e.g., “70%N spline”)

Frequency Response Function of Spline

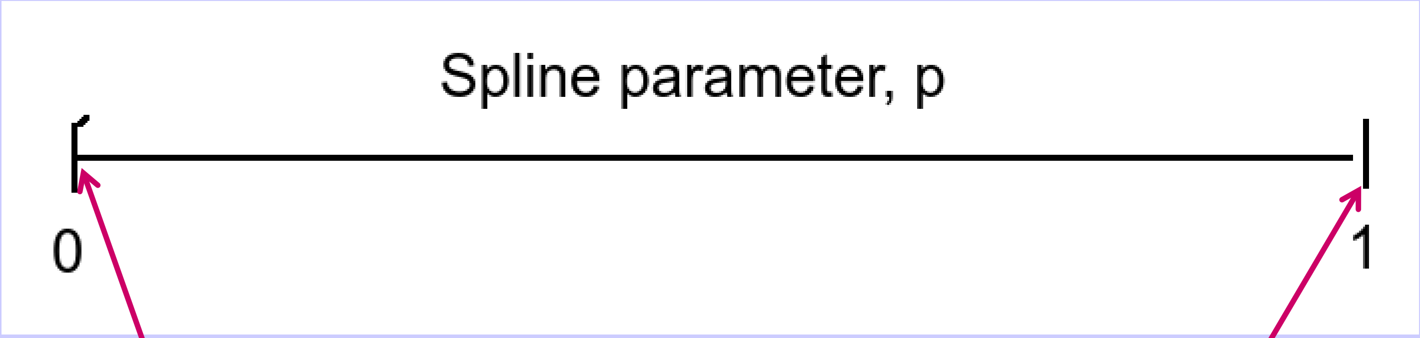


Amplitude of waves– relation to frequency response



var(output)/var(input)=0.25

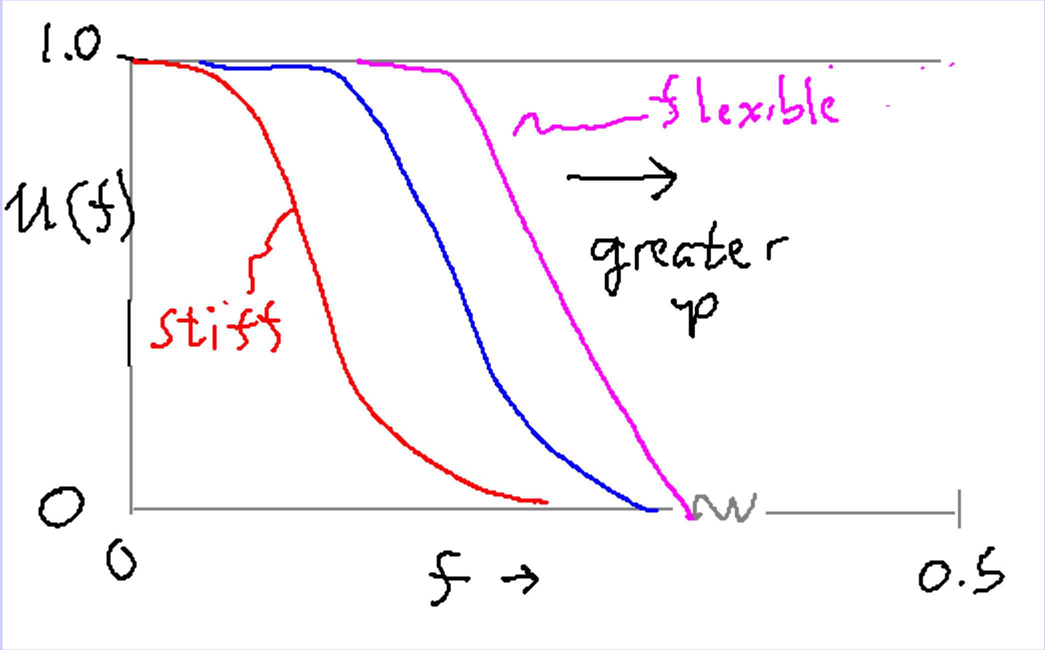
Spline flexibility is controlled though spline parameter



- All weight on smoothness, none on goodness of fit
- Maximum stiffness—approaches straight line

- All weight on goodness-of-fit
- Minimal constraint on smoothness
- Interpolating spline

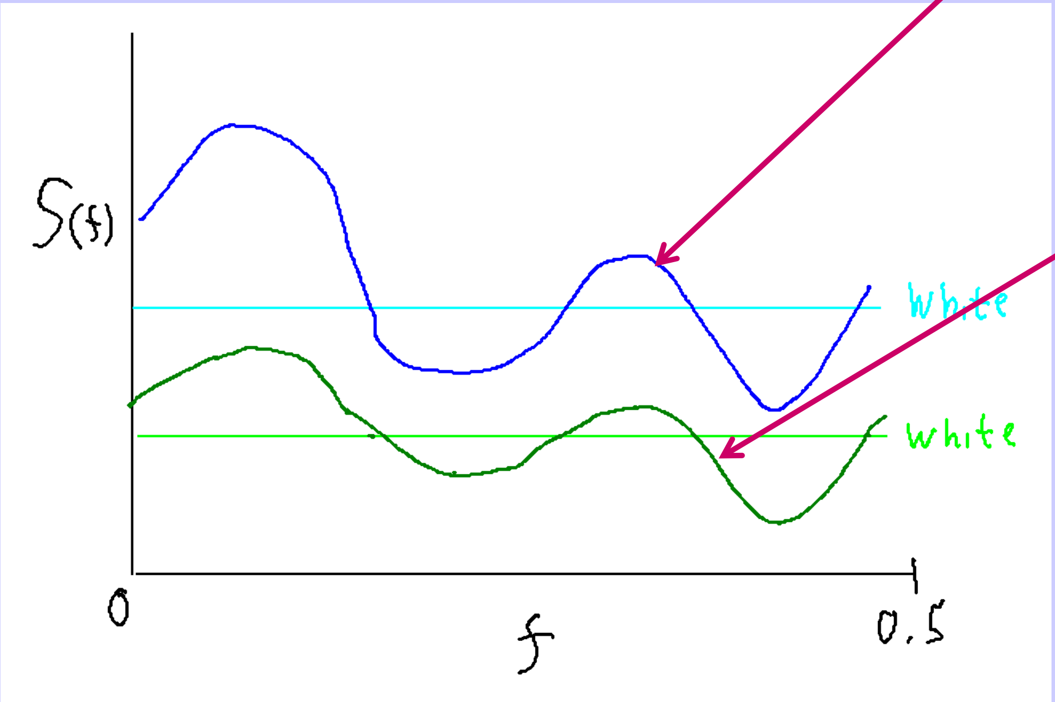
Frequency Response of Spline



$u(f)$ response at frequency f

High $u(f)$: spline closely tracks variations at frequency f; since fitted trend is removed, variance at that and lower frequencies are removed in detrending

Normalized Spectrum



Original spectrum

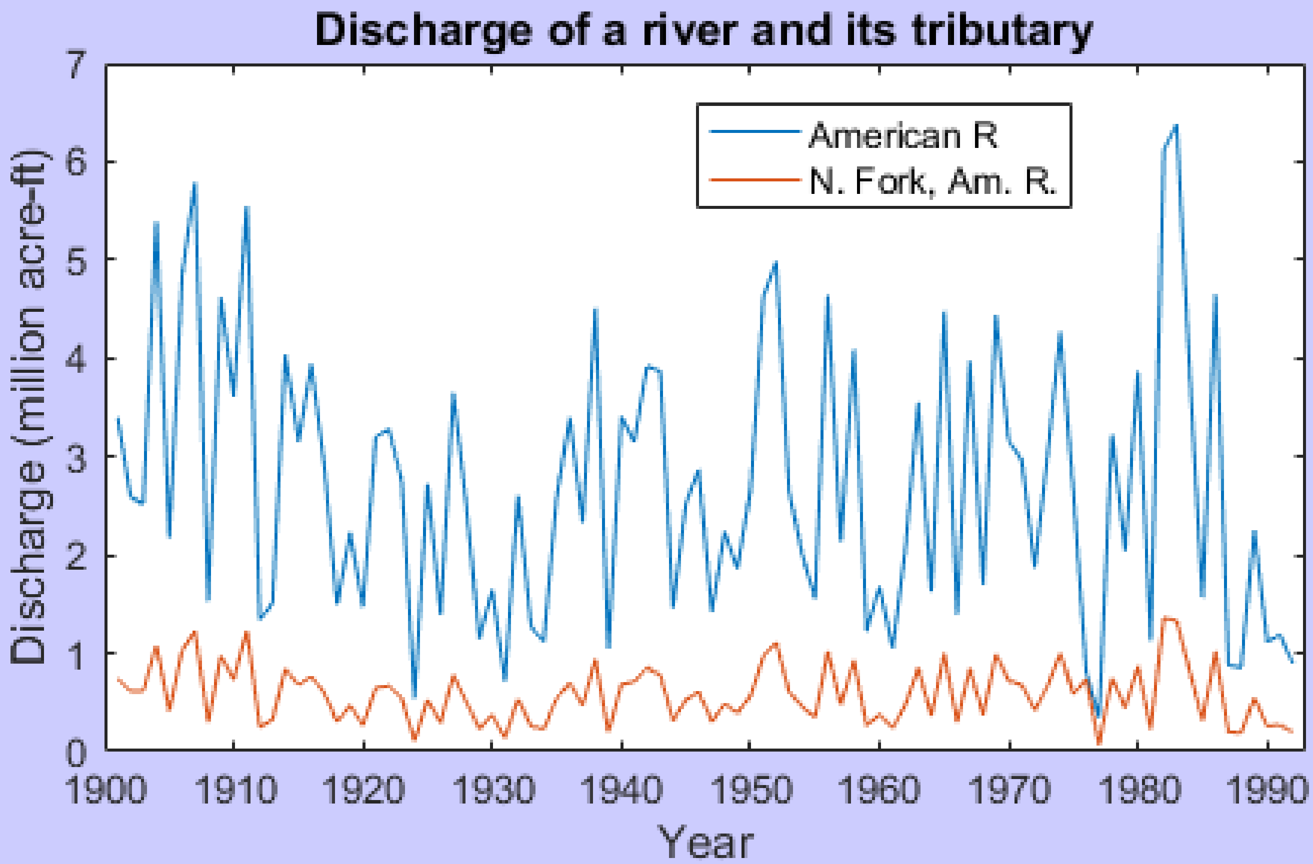
Area α variance of time series

Normalized spectrum

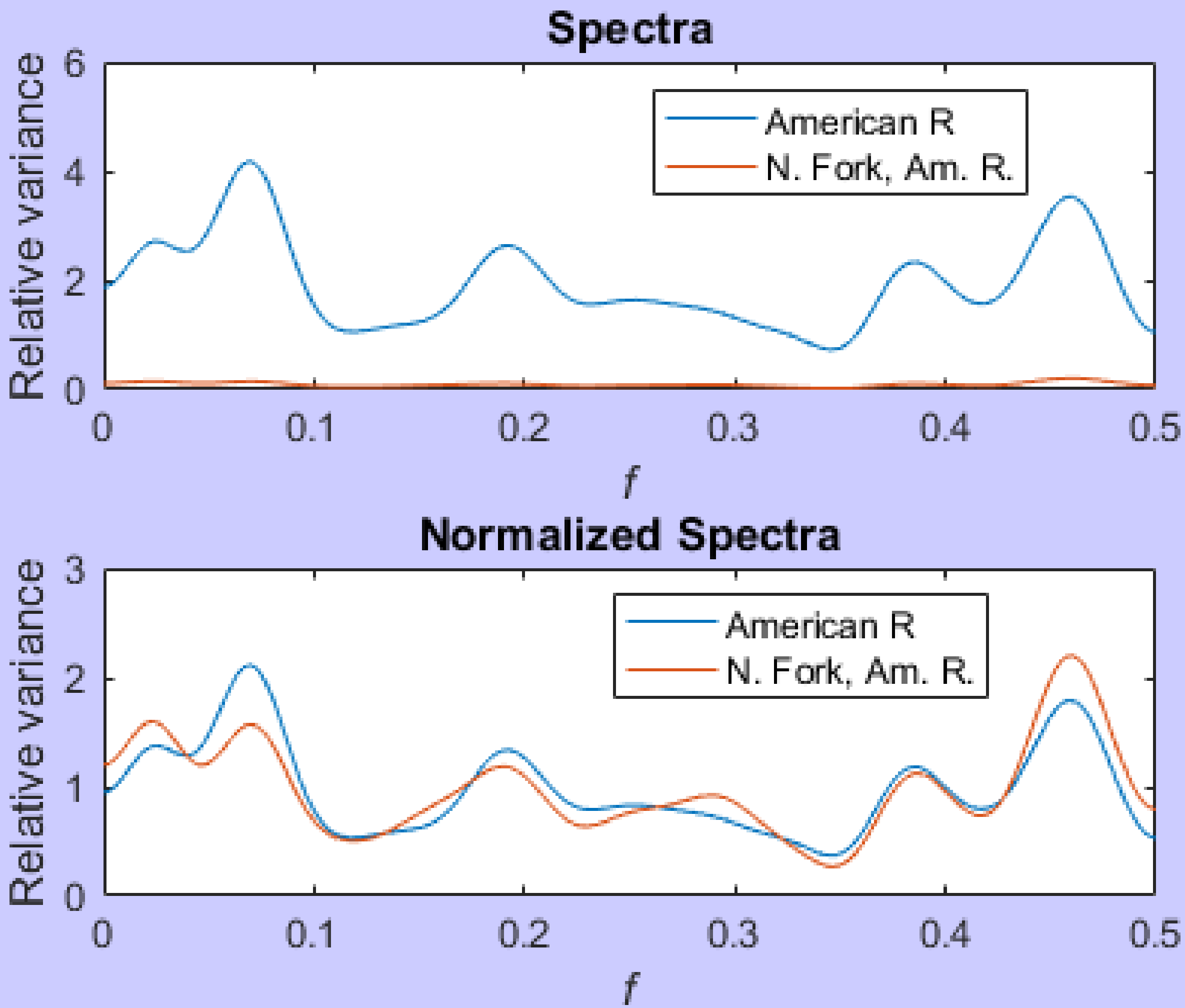
Area = 1

- "Normalize" : scale such that total area under spectrum is 1.0
- Normalized spectrum is also equivalent to spectrum of "z-score transformed" time series
- Simplifies comparison of spectra of time series with different total variances

Time series of two discharge series



Spectrum and normalized spectrum



Demo07a—varying the spline parameter

1. Example using yearly sunspot series
2. Effect on tracking of wavelengths of variation
3. Variance tracked vs variance removed
   (in detrending)

Trial runs of geosa7…